Financial Econometrics

Value at Risk

Gerald P. Dwyer

Trinity College, Dublin

January 2016

Outline

Risk

- What do we mean by risk?
  - Dictionary: possibility of loss or injury
  - Volatility a common measure for assets
  - Two points of view on volatility measure
    - Risk is both good and bad changes
    - Volatility is useful because there is symmetry in gains and losses
- What sorts of risk?
  - Market risk
  - Credit risk
  - Liquidity risk
  - Operational risk
  - Other risks sometimes mentioned
    - Legal risk
    - Model risk

Different ways of dealing with risk

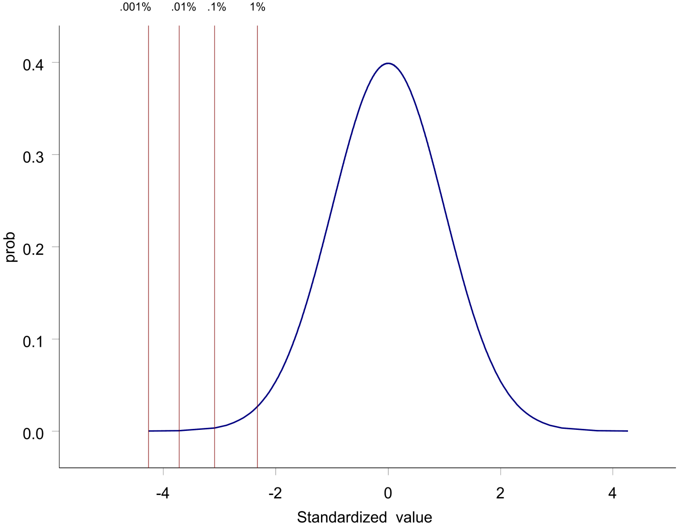
- Maximize expected utility with preferences about risk implicit in the utility function
  - What are problems with this?
- The worst that can happen to you
  - What are problems with this?
- Safety first
  - One definition (Roy): Investor chooses a portfolio that minimizes the probability of a loss greater in magnitude than some disaster level
  - What are problems with this?
  - Another definition (Telser): Investor specifies a maximum probability of a return less than some level and then chooses the portfolio that maximizes the expected return subject to this restriction

Value at risk

- Value at risk summarizes the maximum loss over some horizon with a given confidence level
  - Lower tail of distribution function of returns for a long position
  - Upper tail of distribution function of returns for a short position
    - Can use lower tail if symmetric
  - Suppose standard normal distribution, which implies an expected return of zero
  - 99 percent of the time, loss is at most -2.32634
    - 1 percent of the time, loss is at least -2.32634
  - 99.99 percent of the time, loss is at most -3.71902
    - 0.01 percent of the time, loss is at least -3.71902

Illustration with a normal distribution

Lower tail of distribution



Tail of distribution

- Almost by construction, we care about unusual events, "the tail of the distribution"
- How frequently do we see these events? Suppose daily data
  - 1 percent of the time: 1 day out of every 100
    - Couple of times a year
  - 0.1 percent of the time: 1 day out of every 1,000
    - Once every four years
  - 0.01 percent of the time: 1 day out of every 10,000
    - Once every 40 years
  - 0.001 percent of the time: 1 day out of every 100,000
    - Once very 400 years
- At some point, a question arises whether the data include the risk
  - For example, 2000 to 2006 there was no financial crisis in Ireland
  - Are recent events from the same distribution?

Stress Testing

- One interpretation of stress testing is to go far out in the tail of the distribution
  - What are some pitfalls?
- Another interpretation is to test what happens in some scenario
  - More than a little subjective

Formal definition of VaR

- Value at risk (VaR) is based on the tail of the distribution
  - Let $\Delta V(\ell)$ be the change in the value of assets over the next ℓ periods from t to $t+\ell$
  - Let $F_\ell(x)$ be the cumulative distribution function (CDF) of $\Delta V(\ell)$
  - Let p be the probability of a loss this large or larger
  - Then, for a long position with $\Delta V(\ell) < 0$

$$p = \Pr(\Delta V(\ell) \leq VaR) = F_\ell(x)$$

  - The loss is smaller in magnitude than VaR (i.e. $\Delta V(\ell) > VaR$) with probability $1-p$
  - VaR is the p-th quintile
  - Definition of quintile: For any univariate CDF $F_\ell(x)$ and probability p with $0 < p < 1$, the p-th quintile of $F_\ell(x)$ is

$$x_p = \inf\{x | F_\ell(x) \geq p\}$$

where inf is the operator generating the smallest real number x such that $F_\ell(x) \geq p$.

Example

- VaR

$$p = \Pr\left(\Delta V\left(\ell\right) \leq VaR\right) = F_{\ell}\left(x\right)$$

  - Suppose using a probability of 1 percent
  - Suppose invest $100 and the distribution of value changes is standard normal with zero mean and a variance of one
  - The probability of a loss less than or equal to -$2.33 is 1 percent
  - The value at risk is -$2.33 using this probability
  - This is the same as the 1 percent quintile of the standard normal distribution, which is -2.32634
- How would this differ if the mean were six percent and the standard deviation were one?

RiskMetrics^{TM} overview

- RiskMetrics^{TM} estimation strategy
  - One goal is to estimate relatively few parameters
    - Otherwise estimation error will overwhelm everything else
  - Another goal is to have a fairly objective estimation strategy
    - Few, or better no, subjective decisions made about what parameters to include or exclude
- Technical documents can be found at the course website
- Used IGARCH model until change in 2006
- Problems with IGARCH
  - There is "long memory" in volatility not reflected in IGARCH
    - Autocorrelations of squared returns do not decrease exponentially as indicated for linear and ARCH systems

RiskMetrics$^{\text{TM}}$ overview

- LM-ARCH – long memory ARCH
- Variances at different time scales used and weighted with exponential decay
  - Mean return not zero, especially for stocks and bonds
    - Quantitatively small effects but introduce clear deviations from forecasted volatilities
    - Use autoregressive components from the last two years and estimate mean return over last two years
    - I will suppress this
  - IGARCH(1,1) is

$$
\begin{aligned}
r_t &= \sigma_t \varepsilon_t, \quad \mathrm{E}\,\varepsilon_t = 0, \quad \mathrm{E}\,\varepsilon_t^2 = 1 \\
\sigma_t^2 &= \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1)\left(\sigma_{t-1}\varepsilon_{t-1}\right)^2 \\
0 &< \beta_1 < 1
\end{aligned}
$$

RiskMetrics process

- RiskMetrics setup (2006) is rather more complicated
- Underlying IGARCH(1,1) process with the weight determined by parameters μ_k

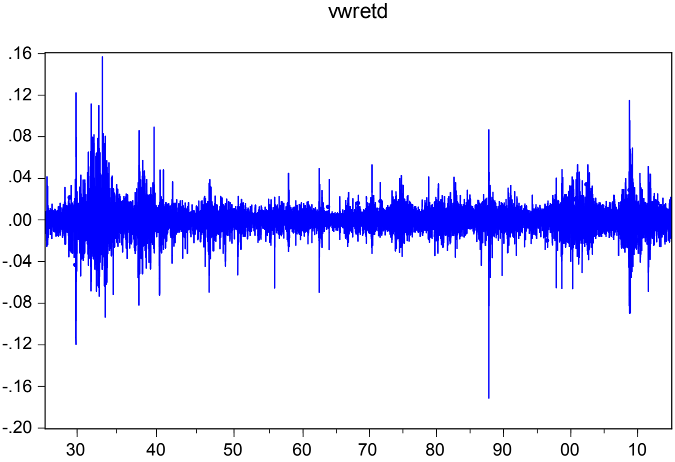
$$
\begin{aligned}
r_t &= \sigma_t \varepsilon_t \qquad (1)\\
\sigma_t^2 &= \sum_{k=1}^{k_{\max}} w_k \sigma_{k,t}^2 \\
w_k &= \frac{1}{C}\left(1 - \frac{\ln \tau_k}{\ln\left(\tau_0\right)}\right) \\
\tau_k &= \tau_1 \rho^{k-1} \quad k = 1, \ldots, k_{\max} \\
\sigma_{k,t}^2 &= \mu_k \sigma_{k,t-1}^2 + \left(1 - \mu_k\right) r_t^2 \\
\mu_k &= \exp\left(-1/\tau_k\right)
\end{aligned}
$$

- Pages 8 and 9 of long document
  - $\tau_k = \tau_1 \rho^{k-1}$ determines weights

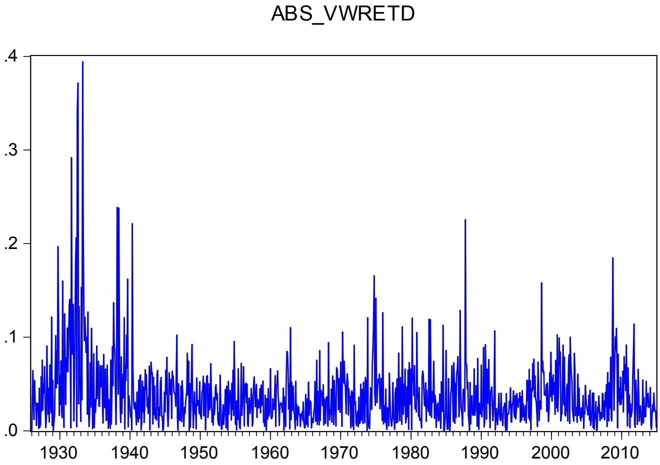
Risk and Financial Crisis of 2007-201?

- Much of losses were not predictable
- Increase in volatility used to forecast continued higher volatility
- Were the events consistent with risk models?
- Finger (2008) argues "yes"
    - Not extraordinary given history of 108 years in the U.S.
    - Would have been extraordinary given estimated parameters over five years of data
    - "Extraordinary" means "not consistent with a model of risk"
- Dowd argues there was a massive failure of VaR models
    - Argues their use by regulatory authorities is particularly "mad"

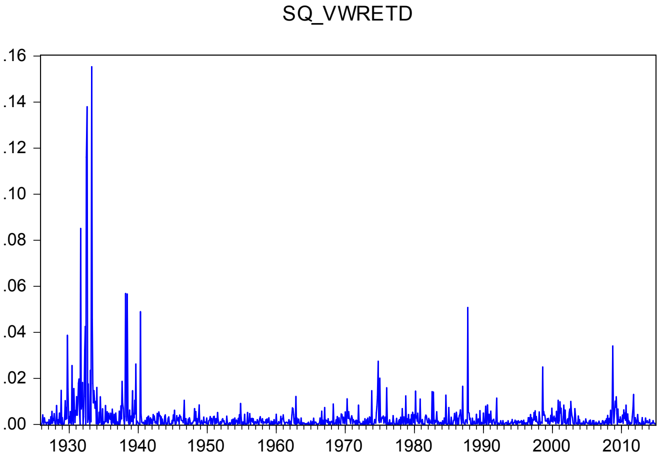
Level and Volatility of CRSP Daily Index Returns



Absolute Value of Daily Returns



Squared Daily Returns



Graphs Summarizing the Distribution of Returns

- Histogram
- Kernel Density
- Cumulative Distribution Function (CDF)

Summary

- Risk has various definitions
- In Finance, risk includes gains and losses
- For Value at Risk, it is the probability of losses that is estimated
- Value at Risk attempts to estimate aspects of the lower tail of the distribution
- GARCH models for recent years are a common way to measure Value at Risk
- Stress tests are another way to estimate risk
  - Scenarios are limited by practicality and imagination

The end

Notes on EViews

- getting density, cdf etc.
  - Open file with series listed by double-clicking on name
  - Go to View and then Graph
  - Choose Distribution as specific and then can choose density, cdf, etc.