

Returns on Stocks



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Returns on Assets

- In one sense, this is a trivial subject
 - What are returns on assets?
 - Do actual returns accord with those predicted by theory?
 - Just compute holding period return

$$h_t = \frac{p_{t+1} + r_t - p_t}{p_t}$$

- p_t is price in period t
 - r_t is any receipt of income while holding asset
- Becomes more complicated in various contexts
 - Survival biases
 - Selection biases more generally
 - Returns over time
 - All income included

Arithmetic or Proportional Return

$$h_t = \frac{p_{t+1} + r_t - p_t}{p_t}$$

- Arithmetic return
- Proportional return

Logarithmic return

$$h_t^l = \ln(p_{t+1} + r_t) - \ln(p_t)$$

- Logarithmic return

Different?

$$\ln(1 + x) \approx x \text{ if } -1 \ll x \ll 1$$

- In English, if x is close to zero relative to one
- Application

$$\ln(1 + h_t) = \ln\left(\frac{p_{t+1} + r_t}{p_t}\right) = h_t^l \approx h_t$$

- Can be justified by
 - Taylor series expansion approximation
 - In continuous time analysis, they actually are identical

Returns and Time

- Mismatch of time period and return period
 - For example
 - Data monthly
 - Returns annual
- Returns over time
 - For example
 - Data and returns monthly
 - Average return over ten-year period

Returns Over Time

- Geometric average annual arithmetic return over ten years

$$\left((1+h_1)(1+h_2)\dots(1+h_{10}) \right)^{1/10} - 1$$

- Proportional return
- Also known as average net return

- Why? Ten-year payoff on a dollar (or euro) is

$$(1+h_1)(1+h_2)\dots(1+h_{10})$$

- Geometric average
- Less investment of one unit

- Average annual log return

$$\frac{\sum_{t=1}^{10} h_t^l}{10}$$

Arithmetic Average Annual Proportional Returns Close?

- Arithmetic average annual proportional return necessarily close to the arithmetic average annual log return?
- NO!
- Arithmetic average of proportional return has undesirable implication
 - Suppose stock prices fall from 100 to 50 and rises back to 100
 - No dividends or other payments
 - Two-year proportional return is zero
 - Arithmetic average log return is zero
 - Arithmetic average proportional return is 25 percent
 - Applying the average for the two years suggests payoff is not 100
 - » Payoff is 100 of course
 - Average suggests payoff is about 150 [or more precisely $(1.25)^2 \cdot 100$]

Estimate of Expected Returns

- Is the geometric average a good estimate of the expected return?
 - Good estimate of the after-the-fact return
 - Ex post
 - Expected is ex ante
 - Some firms have negative average returns
 - Implies negative expected returns?
 - Also, volatility can matter
 - Why?

Brownian Motion

- Simple characterization of price plus income (V)

$$\frac{dV(t)}{V(t)} = \mu dt + \sigma dB(t)$$

- Ito's lemma implies

$$d \ln V(t) = \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dB(t)$$

- Which can be written

$$d \ln V(t) = \alpha dt + \sigma dB(t)$$

- μ is expected return

$$\mu = \alpha + \frac{1}{2} \sigma^2$$

Maximum Likelihood Estimators

$$\hat{\alpha} = \sum_{t=1}^T \Delta V_t / T$$

$$\hat{\sigma}^2 = \sum (\Delta V - \hat{\alpha})^2 / T$$

$$\hat{\mu} = \hat{\alpha} + \frac{1}{2} \hat{\sigma}^2$$

Observation

- Basically it's lognormality of prices (including income) driving this difference between the average log growth rate and the expected growth rate
- This does not normally matter much
- CRSP Value-weighted index 1983 to 2006
 - Mean log growth rate 0.11489
 - Variance of growth rate 0.02323
 - Expected return 0.12650

Sometimes Difference Does Matter

- Personal Computer Firms
 - Mean log growth rate -0.0301
 - Variance of growth rate 0.3043
 - Expected return 0.1221

**Personal Computer Companies and Stock Final Values
December 1983 through December 2006
Individual Companies**

Expected Return for Personal Computer Firms	Mean Log Return	Variance of Log Return	Expected Return for CRSP Value-Weighted Index	Expected Return for CRSP Equally-weighted Index	Kolmogorov-Smirnov Test	Bera-Jarque Test
0.1220	-0.0301	0.3043	0.1265	0.1375	0.2016	1.9051
0.0318	0.0316	0.2908 0.3188	0.0370	0.0441	(0.0127)	(0.3857)

	<u>Date</u>			<u>Cumulative Value (\$)</u>						
	First	Last	Annual Return	Entire Period	1983-1987	1987-1991	1991-1995	1995-1999	1999-2003	2003-2006
<i>Average</i>			-0.0907	14.5331	1.5872	0.9754	1.7475	5.8287	0.6201	1.8913
<i>Median</i>			-0.0108	0.8505	1.3499	0.7171	1.1606	2.5521	0.4650	0.7384
<i>Standard Deviation</i>			0.2779	52.2216	1.3452	1.0509	1.5724	11.7987	0.4913	2.7104
<i>CRSP Value-Weighted Index</i>			0.1218	14.0473	1.5952	1.8950	1.6386	2.4189	0.8319	1.4094
<i>CRSP Equal-Weighted Index</i>			0.1286	16.1592	1.0976	1.5790	1.9894	1.8403	1.6681	1.5268

<u>Company</u>	<u>Date</u>		<u>Annual Return</u>	<u>Entire Period</u>	<u>1983-1987</u>	<u>1987-1991</u>	<u>1991-1995</u>	<u>1995-1999</u>	<u>1999-2003</u>	<u>2003-2006</u>
Dell Computer Corporation	Jun-88		0.3498	256.9216		2.7333	4.0537	47.1336	0.6663	0.7384
Zeos International	Sep-89	Mar-95	0.2075	2.8209		4.2985	0.6563			
NCR Corporation		Aug-91	0.1997	4.0392	2.1672	1.8638				
Compaq Computer		Apr-02	0.1916	24.8841	4.4300	0.9526	5.4597	2.8356	0.3809	
Apple Computers Incorporated			0.1603	30.5241	3.4636	1.3962	0.5929	3.2255	0.4157	7.9401
Canon Incorporated		Dec-97	0.1207	4.9291	1.2977	1.7874	1.6478	1.2896		
Hewlett Packard Company			0.1150	12.2241	1.4040	1.0121	3.0823	2.8152	0.5328	1.8608
IBM			0.0749	5.2609	1.0784	0.9141	1.1524	4.8691	0.8803	1.0804
Digital Equipment Corporation		May-	0.0300	1.5313	3.7500	0.4093	1.1606	0.8597		
Advanced Logic Research	Apr-90	Jun-97	0.0204	1.1557		0.7170	0.6316	2.5521		
NEC Corporation			-0.0003	0.9932	2.6366	0.6730	1.3167	2.0516	0.3164	0.6548
Inmac Corporation	Oct-86	Dec-95	-0.0004	0.9962	1.4020	0.1921	3.7000			
Unisys Corporation			-0.0211	0.6125	2.3282	0.1384	1.3333	5.8068	0.4650	0.5280

Personal computer firms

- Personal computer firms μ 0.1220 per year
– α -0.0301 σ^2 0.3043
–
- CRSP equally-weighted μ 0.1253 per year
- CRSP value-weighted μ 0.1357 per year

Airlines and airplane manufacturers 1925 to 1940

- Airlines μ 0.1728 per year
– α -0.0325 σ^2 0.4105
- Airplane manufacturing μ 0.3315 per year
– α 0.0837 σ^2 0.4957
- CRSP equally-weighted μ 0.1504 per year
– α 0.0607 σ^2 0.1795
- CRSP value-weighted μ 0.0781 per year
– α 0.0297 σ^2 0.0968

Automobile manufacturers

1912 to 1928

- Auto manufacturing μ 0.3481 per year
 - α 0.0885 σ^2 0.5193
- Cowles index μ 0.1161 per year
 - α 0.1098 σ^2 0.0125
- Data
 - Stock prices from *New York Times*
 - Dividends from Moody's and similar sources

Railroads Traded on the NYSE

1855 to 1870

- Railroads traded on NYSE μ 0.0801 per year
 - α 0.0512 σ^2 0.0579
- Goetzmann index μ 0.1734 per year
 - α 0.1334 σ^2 0.0800
- Data
 - Prices from *Hunt's Merchants Magazine*, *Bankers' Magazine* and *New York Times*
 - Dividends from Moody's

Summary table

Industry	Industry expected return	Index expected return
PCs	0.1216	0.1253
Airlines	0.1728	0.1504
Airplane Manuf.	0.3316	0.1504
Autos	0.3481	0.1161
NY Railroads	0.0801	0.1734

Good or Bad Investment?

- Sharpe ratio is one way to evaluate portfolios
- Consider a portfolio of firms
 - Haven't we already done that?
 - No

$$SR = \frac{\mu - R^f}{\sigma}$$

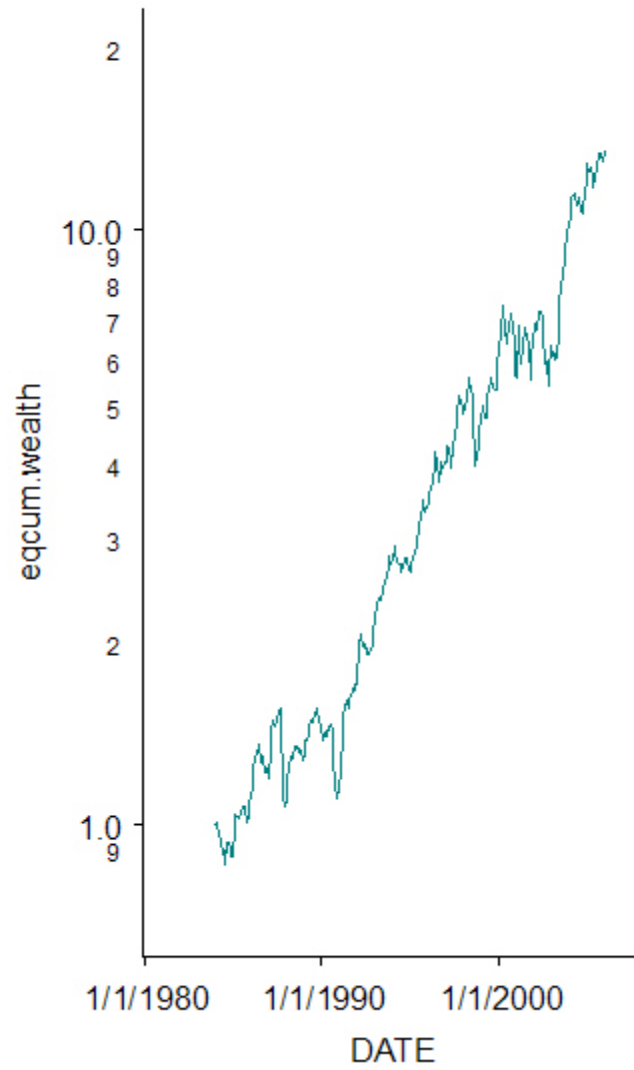
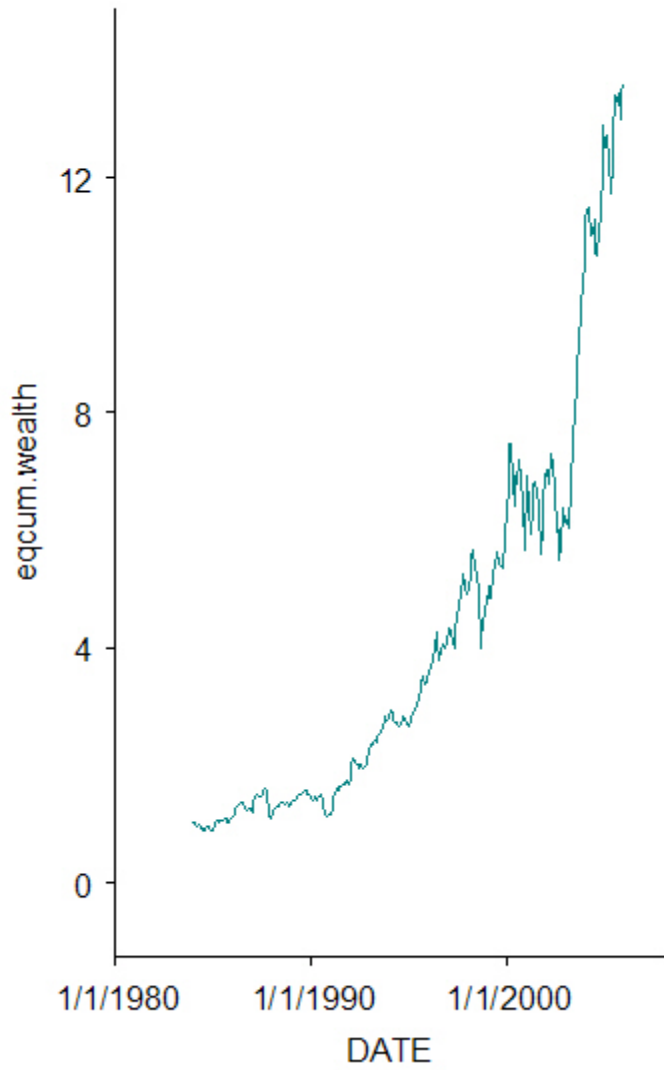
Selection of Firms

- For personal computer firms
 - 14.53 average cumulative value
 - A particular kind of portfolio
 - Equally invested in each firm while it exists
 - A firm's fraction of total is not invested in anything when a firm does not exist

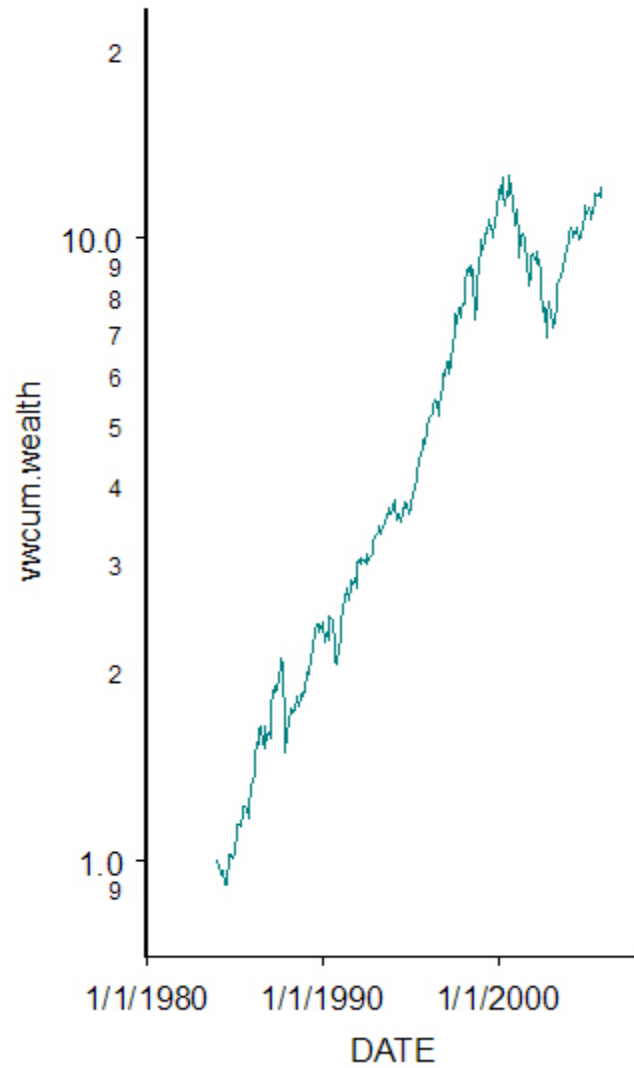
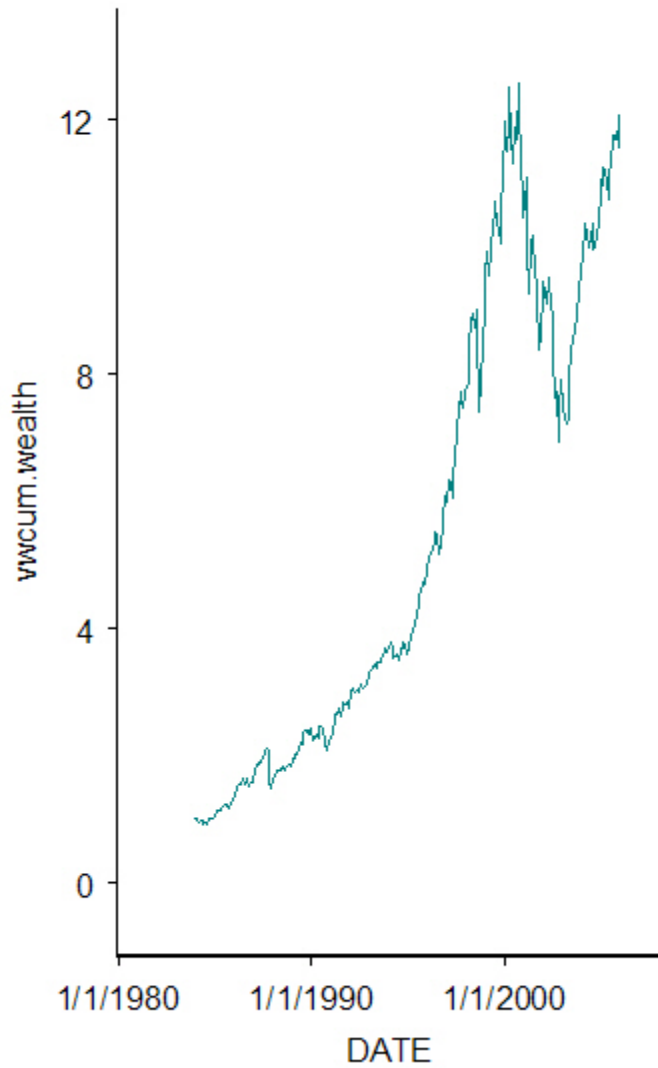
More Interesting Portfolio

- Equally weighted of existing firms at any time
 - Invested in firms at all times
- Value weighted might be better, but data not available historically

Equally Weighted CRSP Index
Linear and Log Scale



Value Weighted CRSP Index
Linear and Log Scale



Personal Computer Index



Sharpe Ratios

Industry	Dates	Industry Sharpe Ratio	Market Equally Weighted Sharpe Ratio	Market Value Weighted Sharpe Ratio
PCs	December 1983 to December 2006	0.4516	0.4780	0.5014
Airlines	December 1925 to December 1940	0.1949	0.3627	0.2083
Airplane Manufacturers	December 1925 to December 1940	0.5443	0.3627	0.2083
Autos	December 1912 to December 1928	0.4821	–	0.6392
NYSE Railroads	September 1851 to December 1870	0.0084	1.1256	–

Returns

- Sharpe ratio indicates that holding a portfolio of stocks in new industries is not a good deal in terms of mean return and variance
- Doesn't mean that they're not sensible as part of a diversified portfolio
- Doesn't mean that they're not interesting given different objectives
 - Skewness of returns may matter

Selection Issues

- In this study, focused on “successful” new industries
 - Are there failed new “industries”?
- Why not look at telephone?
 - Telegraph?
- What are the implications for generalizing to all “new industries”?

Selection Issue and Averaging

- Can average returns different ways
 - Across firms
 - Across time periods for each firm
 - Use each return for each firm for each period as an observation
 - Firms did not exist for same number of periods
 - How to average depends on question asked
 - “Selection bias” is really a problem of generalizing a result from a “sample” to a population that is not the population from which this sample was drawn
 - With a difference between the two populations that matters for the statistical question being asked