Monetary Economics
Measuring Asset Returns

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Fall 2015
Readings

• Readings this lecture, Cuthbertson Ch. 9
• Readings next lecture, Cuthbertson, Chs. 10-13
Measuring Asset Returns

• Outline
• Calculating returns
• Equity risk premium
• Statistics for summarizing data
  – Moments
  – Measures of association
Measuring Returns on Assets

• Measuring asset returns might seem relatively trivial
  – It is trivial in a way
  – It is rather involved in a way
Measuring Returns on Assets

• Measuring asset returns might seem relatively trivial
  – It is trivial in a way
  – It is rather involved in a way

• What return?
  – Nominal or real?
  – Dividends reinvested or not?
  – Proportional return, compounded return, continuously compounded return
  – Average return: arithmetic mean versus geometric mean
Straightforward Measure of Return

• Return for last period
  \[ \text{Return}_t = \frac{\text{Cash flows received}_t - \text{Cash flows paid out}_{t-1}}{\text{Cash flows paid out}_{t-1}} \]

• Percentage Terms
  \[ \text{Return}\%_t = 100 \cdot \text{Return}_t \]

• Holding period return
• Backward looking measure
• Called \textit{ex post} return
Ex Ante Return

• A forward looking measure of return for one period is

\[
\text{Return Forward}_t = \frac{\text{Expected cash flows received}_{t+1} - \text{Cash flows paid out}_t}{\text{Cash flows paid out}_t}
\]

• Called *ex ante* return

• The future return must be expected or anticipated return
Nominal and Real Rate

• Nominal rate on a discount security for one period
  – Pay $95 now and receive $100 a period from now
    \[
    \frac{\$105 - \$100}{\$100} = .05 \text{ or } 5.00 \text{ percent}
    \]

• Real rate on the discount security
  – Suppose that the price of a tank of gas increases 2 percent, from $50 to $51
  – What is the real interest rate?
    • The interest rate in terms of tanks of gas here
    • Formula is
      \[
      \text{Real interest rate} \approx \text{Nominal interest rate} - \text{inflation rate}
      \]
Real Rate on the Discount Security

• Pay $100 now for the discount security and get $105 a year from now
• A tank of gas costs $50 now
• A tank of gas costs $51 a year from now
• $100 today buys two tanks of gas
• $105 a year from now buys 2.0588 tanks of gas
• Interest rate in terms of tanks of gas is
  \[
  \frac{2.0588 - 2}{2} = 0.0294 \text{ or } 2.94\%
  \]
  – Approximately 3 percent
One-period Measure of Return

• This return is the *ex post holding period return*

\[
\text{Return}_t = \frac{\text{Cash flows received}_t - \text{Cash flows paid out}_{t-1}}{\text{Cash flows paid out}_{t-1}}
\]

• This return is the *ex ante holding period return*

\[
\text{Return Forward}_t = \frac{\text{Expected cash flows received}_{t+1} - \text{Cash flows paid out}_t}{\text{Cash flows paid out}_t}
\]
Return Over Several Periods

• Suppose a security has prices in three years

$$P_0 = 100, P_1 = 110, P_2 = 105$$

• Cumulative values are 110, 105

• Holding period returns are

  10 percent and -4.5454... percent per year

• What is typical return?
  – Arithmetic mean is 2.7272... percent per year

• If this average return is applied to initial $100, get

  $100 \cdot (1.02727)^2 = $105.5289 \neq $105$
Better Measure of Average Return

• *Geometric mean* is a better measure of typical return
  – Better because reflects variability of return and effect on final cumulative value

• Rather than taking arithmetic average of returns, take geometric average
Geometric Average Return

• Security has prices in three years
  \[ P_0 = 100, P_1 = 110, P_2 = 105 \]

  \[ g = \left( \frac{105}{100} \right)^{\frac{1}{2}} - 1 = 0.0247 \text{ or } 2.47 \text{ percent per year} \]

• The geometric mean is the average holding period return with annual compounding which would generate the final value received
Geometric Average Return

• Security has prices in three years

\[ P_0 = 100, \ P_1 = 110, \ P_2 = 105 \]

\[ g = \left( \frac{105}{100} \right)^{\frac{1}{2}} - 1 = 0.0247 \text{ or } 2.47 \text{ percent per year} \]

• The geometric mean \( g \) is the average holding period return with annual compounding which would generate the final value received

• Holding period returns are

10 percent and -4.5454...

\[ 1.1 \times 0.9545... = 1.05 \]
Geometric Average Return in General

• For an investment lasting $T$ years, the geometric average annual return is

$$g = \left( \frac{W_T}{W_0} \right)^{\frac{1}{T}} - 1$$

– where $W_0$ is the initial value and $W_T$ is the final value
Overall Market Dividends Reinvested
December 31, 1984 to December 31, 2014

vwcrspd_84
Continuously Compounded Returns

• Also called log returns
  – Natural logarithm

• Log returns often more convenient
  – Reduce size of extreme returns
  – Multiplication becomes addition
  – Multi-period returns simple to calculate
  – Initial value of $100 and final value of $110 a year from now

  9.531 percent
Table 1: Compounding frequencies

<table>
<thead>
<tr>
<th>Compounding frequency</th>
<th>Value of $100 at end of year (r = 10% p.a.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually (q = 1)</td>
<td>110</td>
</tr>
<tr>
<td>Quarterly (q = 4)</td>
<td>110.38</td>
</tr>
<tr>
<td>Weekly (q = 52)</td>
<td>110.51</td>
</tr>
<tr>
<td>Daily (q = 365)</td>
<td>110.5155</td>
</tr>
<tr>
<td>Continuously compounding</td>
<td>110.5171</td>
</tr>
<tr>
<td>TV = $100e^{0.1(1)} (n = 1)</td>
<td></td>
</tr>
</tbody>
</table>

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Variability of Returns

• With daily data, easy to compute daily standard deviation of returns
  – For CRSP index, this is 0.01066
  – In percentage terms, this is about 1.1 percent per day

• Monthly or annual basis
  – Simple way – multiply by square root of number of observations
  – Monthly standard deviation
    0.011094 * square root(30) = 0.060764
    6 percent per month
  – Annual standard deviation
    0.010660 * square root(252) = 0.169
    16.9 or 17 percent per year
Equity Risk Premium

• Does the low average real return on stocks since December 31, 1999 mean that the real return will be equally low in the future?
  • 4.7 percent per year nominal
  • Inflation 2.27 percent per year
  – Real return has been quite high lately
    • Nominal return since December 1, 2008 is 16.9 percent per year
    • Inflation rate is 1.9 percent per year

• What is a reasonable inference from the data?
## Returns Over Various Periods

<table>
<thead>
<tr>
<th>Date</th>
<th>CRSP_d</th>
<th>years</th>
<th>Ann Avg return</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31/1999</td>
<td>11.714</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12/31/2008</td>
<td>9.113</td>
<td>9</td>
<td>-0.027512525</td>
</tr>
<tr>
<td>12/31/2014</td>
<td>23.222</td>
<td>6</td>
<td>0.168708964</td>
</tr>
<tr>
<td>Total</td>
<td>23.222</td>
<td>15</td>
<td>0.046677673</td>
</tr>
</tbody>
</table>
Figure 4: Inference: Mean and std dev: annual averages (post 1947)

- Standard deviation of returns (percent)
- Average Return (percent)

Values: Government Bonds, Corporate Bonds, T-Bills, S&P500, Value weighted, NYSE, Equally weighted, NYSE

- Smallest "size sorted" decile
- Largest "size sorted" decile
- Individual stocks in lowest size decile

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Past and Future

• Sometimes we just want to summarize data
  – What has happened?
• Often want to draw inferences about what is likely to happen in the future
  – Statistics: often want to draw inferences about population from a sample
• In contexts where looking at time series, often want to make predictions about the future
  – Everything is different all the time
  – Everything is the same all the time
Differences Across Firms

• The differences in cost of equity capital across firms are entirely due to differences in beta

\[ E R_s = E r + \beta (E R_m - E r) \]

• Riskfree rate is 2.20 percent per year and risk premium for the market is 5.6 percent

<table>
<thead>
<tr>
<th>Firm</th>
<th>Beta</th>
<th>Risk premium</th>
<th>Expected return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amazon</td>
<td>1.35</td>
<td>7.56</td>
<td>9.76</td>
</tr>
<tr>
<td>Whole Foods</td>
<td>1.32</td>
<td>7.39</td>
<td>9.59</td>
</tr>
<tr>
<td>Ford</td>
<td>1.37</td>
<td>7.67</td>
<td>9.87</td>
</tr>
<tr>
<td>Krispy Kreme</td>
<td>2.41</td>
<td>13.50</td>
<td>15.70</td>
</tr>
<tr>
<td>Duke Energy</td>
<td>0.44</td>
<td>2.46</td>
<td>4.66</td>
</tr>
</tbody>
</table>
Estimates of Beta

• Are these estimates of beta plausible for the future?
Summarizing Data is a Solid Start

• Time series graphs
• Histogram
CRSP Index

Series: VWRETD
Sample 12/31/1925 12/31/2014
Observations 23534

Mean 0.000411
Median 0.000770
Maximum 0.156838
Minimum -0.171349
Std. Dev. 0.010660
Skewness -0.120265
Kurtosis 19.87127
Jarque-Bera 279169.8
Probability 0.000000
Normal Distribution

X

Density

-5 -4 -3 -2 -1 0 1 2 3 4 5

0.0 0.1 0.2 0.3 0.4 0.5
CRSP Index

Series: VWRETD
Sample 12/31/1925 12/31/2014
Observations 23534

Mean       0.000411
Median   0.000770
Maximum  0.156838
Minimum -0.171349
Std. Dev.   0.010660
Skewness  -0.120265
Kurtosis   19.87127
Jarque-Bera  279169.8
Probability  0.000000
Normal Distribution

Series: X
Sample 1 100000
Observations 100000

Mean: -0.002577
Median: 0.002484
Maximum: 4.047115
Minimum: -4.422242
Std. Dev.: 1.000132
Skewness: -0.009953
Kurtosis: 2.989377
Jarque-Bera: 2.121066
Probability: 0.346271
Series: RET
Sample 5/15/1997 12/31/2014
Observations 4436

Mean       0.001977
Median   0.000000
Maximum  0.344714
Minimum -0.247661
Std. Dev.   0.041282
Skewness   0.984702Kurtosis   11.51157
Jarque-Bera  14107.46
Probability  0.000000
Moments

• Mean
  – Arithmetic average
• Range often useful
• Variance and standard deviation
• Skewness
• Kurtosis (or excess kurtosis)
Generalizations about Stock Prices

• Typically skewed to the left
• More certainly, stock prices have *fat tails*
  – A distribution has fat tails if the upper and lower ends of the distribution have more observations than a normal distribution
Association of Series

- Linear association can be measured by covariance, correlation and regressions
- Covariance for $R_A$ and $R_B$ for a set of data with $n$ observations is
  \[
  \hat{\sigma}(R_A, R_B) = \frac{\sum_{t=1}^{n} (R_{A,i} - \bar{R}_A)(R_{B,i} - \bar{R}_B)}{n-1}
  \]
  - $\bar{R}_A$ is the mean of the returns on stock A and $\bar{R}_B$ is the mean of the returns on stock B
Covariance

• Covariances are useful but not so informative by themselves
• Covariance between Amazon and CRSP is 0.000235
• Big or small?
  – Not obvious what to compare this number to
  – Worse, if measured returns in percentage terms, the covariance would be 2.35
    • Magnitude depends on units of variables
The correlation between $R_A$ and $R_B$ for a set of data with $n$ observations is

$$
\hat{\rho} = \frac{\hat{\sigma}(R_A, R_B)}{\hat{\sigma}(R_A)\hat{\sigma}(R_B)}
$$

where $\hat{\sigma}(R_A, R_B)$ is the covariance between $R_A$ and $R_B$ and $\hat{\sigma}_A$ and $\hat{\sigma}_B$ are the standard deviations for $R_A$ and $R_B$.

- Big advantage: Varies between 1 and -1
- 0.45 for Amazon and CRSP since Amazon’s IPO
Regression

• A regression equation between $R_A$ and $R_B$ is

$$R_{A,t} = \alpha + \beta R_{B,t} + \varepsilon_t$$

where $\beta$ is a measure of the “effect” of $R_B$ on $R_A$

– The coefficient $\alpha$ is a constant term that reflects nonzero mean values and $\beta$ is a residual term to reflect other factors $\varepsilon_i$

– The coefficient beta in CAPM is called “beta” because it is a regression coefficient

– $\hat{\beta}$ is computed from

$$\hat{\beta} = \frac{\hat{\sigma}(R_A, R_B)}{\hat{\sigma}^2(R_B)}$$

– 1.46 for Amazon and CRSP
Regression coefficients

• $\hat{\beta}$ depends on the units of variables
  – Supposed to measure “effect” so that is what we want

• Correlation is not causation
Summing Up

• Holding period return simplest and common
• Returns require care with compounding
• Ex ante returns versus ex post returns
• Geometric average of returns generally better
• Equity risk premium in the past and future
Summing Up

• Summarizing data
  – GRAPHS

• Statistics
  – Moments
  – Measures of association