NONLINEARITY AND BUSINESS CYCLES

Cora Barnhart
Florida Atlantic University
Federal Reserve Bank of Atlanta

Gerald P. Dwyer, Jr.
Federal Reserve Bank of Atlanta

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Abstract

The evidence concerning nonlinearities in macroeconomic series is mixed. In this paper, we pursue an insight due to Hicks (1950, 1982) that asymmetric adjustment costs associated with recessions will be reflected in many series, not just the obvious output series. We examine output and related series, financial and monetary series, and nominal series for evidence of nonlinearity. The results indicate that predictable changes in the series associated with recessions are important. Simple threshold autoregressions using the unemployment rate as the threshold variable are consistent with the nonlinearity being due to related business-cycle behavior in recessions.

I. INTRODUCTION

Contractions in economic activity are steeper and shorter than the subsequent recovery and expansion. This common-place observation can be illustrated by the unemployment rate, for example, which increases rapidly in recessions and falls slowly in the subsequent expansion. A number of economists including Hicks (1950, 1982) and Friedman (1993) have suggested that so-called capacity constraints are important for understanding business cycles. Capacity constraints can be thought of more generally as asymmetric adjustment costs. Empirical evidence indicates that at least a few macroeconomic time series such as the unemployment rate behave differently in contractions than in recoveries or expansions (e.g. Beaudry and Koop 1993; Montgomery, Zarnowitz, Tsay and Tiao 1998). Despite this apparent aspect of some data, statistical economic models of the aggregate economy rely almost exclusively on linear models such as vector autoregressions.

While there have been numerous tests for nonlinearity in various economic series, the research to date yields inconsistencies.[1] DeLong and Summers (1984) find asymmetry in unemployment but not industrial production or GNP. Brock and Sayers (1988) find nonlinearity in employment and industrial production but not quarterly GNP and investment. Pfann (1991) detects nonlinearity in seven quarterly unemployment rates. Balke and Fomby (1994) examine fifteen quarterly production and employment series and conclude that nonlinearity disappears once

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[1] Nonlinear analysis of financial series has used a variety of tests and found more consistent results. Hinich and Patterson (1985) apply the bispectral test from daily returns of ninety-five stocks. Marsh (1985) examines daily returns of stocks and two indices using this test. Both studies find results inconsistent with Gaussian processes. Ashley and Patterson (1989) find evidence inconsistent with linearity in percentage changes of daily returns in the CRSP stock index. Hsieh (1991) uses the BDS test and a third-order unconditional moments test and finds evidence inconsistent with independent and identically distributed weekly returns of the CRSP stock index. Cao and Tsay (1992) detect nonlinearity with the Tsay F, augmented F, and BDS tests. Rothman (1994) examines weekly returns and finds time irreversibility.

these series are adjusted for what they identify as outliers. More recent contributions include those by Ramsey and Rothman (1996), Verbrugge (1997), and Montgomery *et al.* (1998).

Most research to date on macroeconomic series relies on annual or quarterly data and pays little attention to series other than the unemployment rate, output measures and stock prices.[2] Replacing quarterly or annual data with monthly series increases the likelihood of finding asymmetries. Not only are business cycles likely to be more distinct in higher frequency data, but many tests for nonlinearity explicitly or implicitly rely on higher-order moments that require more observations to be estimated precisely.

The purpose of this paper is to examine the generality of nonlinearity in aggregate economic series and determine whether that nonlinearity can be associated with asymmetric adjustment costs. As Hicks (1950, 1982) points out, asymmetries generally will leave a trace in variables such as interest rates and real money balances as well as variables directly affected by asymmetric adjustment costs. Section two defines a linear time series, presents the data series, and discusses the estimation of linear filters. This section discusses the implications of tests for unit roots and presents linear filters for the serial correlation in the series. The usefulness of a filter for Autoregressive Conditional Heteroskedasticity (ARCH) for our series also is examined. Section three presents the results of a battery of tests for asymmetry and a particular variety of asymmetry, time reversibility, and some commonly used tests for nonlinearity. This section also presents tests whether estimation of ARCH parameters, which dramatically reduces other evidence of

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[2] One notable exception is Mizrach (1994), who compares the power of the BDS and U-statistics in detecting asymmetry on monthly series for France, Germany, and Italy, including real M3, indices for industrial production and labor cost, the GDP deflator, and the unemployment rate.

nonlinearity, adequately characterizes the data. The final section uses changes in the unemployment rate to create common thresholds and examines the informativeness of a common business-cycle component of the series.

II. ESTIMATION OF LINEAR MODELS

In this section, we discuss the series that we use and the filters we use to remove linearity and a particular form of nonlinearity, autoregressive conditional heteroskedasticity.

Linearity and Nonlinearity

What does it mean to characterize a series as linear? Under fairly general conditions, every time series has a Wold Representation

(1) $x_t = a_t + b(L)e_t,$

where a_t is deterministic, *b(L)* is a polynomial in the lag operator L with $\sum_{i=0}^{\infty} b_i^2 < \infty$ and e_t is a zero mean, constant variance, serially uncorrelated process. Suppose for simplicity that the deterministic part of $\{x_t\}$ is zero. This representation with uncorrelated innovations leaves much of the probability distribution of $\{x_t\}$ unspecified, a point made by Priestley (1988, pp. 23-23.)

If the innovations are independently and identically distributed (i.i.d.), there exists the seemingly similar but more restrictive and better defined representation

(2) $x_t = c_t + d(L)\varepsilon_t,$

where c_t is deterministic, *d(L)* is a polynomial in the lag operator L that also is square summable and ε_t is a zero mean, independent and identically distributed (i.i.d.) process. With these i.i.d. innovations, the conditional expectation is a linear function of the innovations. If this moving average representation is invertible, the predictable part of the stochastic process can be represented by the linear autoregression

(3) $E[x| x_{t-1,} x_{t-2}, \ldots.] = \alpha_t + \beta(L) x_{t-1}$.

The converse does not hold: it is possible for the conditional expectation to be a linear function of past observations even though the innovations in (1) are not i.i.d. Combining a linear autoregression with autoregressive conditional heteroskedastistic (ARCH) innovations is such a process that commonly is estimated using economic data. An ARCH representation has dependent innovations but the conditional expectation is a linear function of past observations.

One less familiar aspect of a time series is worth mentioning here: time reversibility. The basic idea of time reversibility can be illustrated by the graph of the civilian unemployment rate in Figure 1. The shaded areas are recessions as defined by the National Bureau of Economic Research (National Bureau of Economic Research 1998.) This figure indicates that the unemployment rate increases rapidly in recessions and decreases gradually during the subsequent expansions, a pattern recognized by among others, e.g. Montgomery *et al.* (1998.) If time were reversed, the effect of which can be seen by flipping the page over, this graph would show slow increases and rapid decreases. This is an example of a series that is *not* time reversible.

One formal definition of time reversibility states that the joint probability distribution of *l* observations is invariant to reversing the origin and ordering of time.[3] Weiss (1975), Hallin, Lefevre and Puri (1988) and subsequent remarks by Tong (1990) show that a linear moving-average representation with i.i.d. innovations is time reversible if and only if the innovations are

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[3] There are other definitions (Whittle 1975; Kelly 1979.)

Gaussian. Nonetheless, a linear moving-average representation with uncorrelated but possibly dependent observations can be time reversible. A linear autoregressive representation that is time reversible need not be Gaussian.

This analysis motivates our empirical strategy. We examine residuals from linear autoregressions for evidence of dependence. As is commonly done, we assume that the moving-average representation is invertible and rely on a finite-order autoregression to approximate the infinite-order autoregression.

Empirical Analysis

Our empirical analysis includes monthly measures of output and related series, financial and monetary series, and nominal variables. The set of series is intended to include measures of these aspects of aggregate economic activity. The variables associated with output are industrial production, real disposable personal income, real retail sales, total employment, manufacturing employment, the unemployment rate, the inventory-sales ratio, total capacity utilization in manufacturing, the real value of orders for durable goods and the real value of orders for industrial machinery. The financial and monetary series are the federal funds rate, the nominal 90-day Treasury Bill rate, the six-month commercial paper rate, the ten-year Treasury bond rate, the real 90-day Treasury Bill rate, the real S&P 500 index and the real quantity of money measured by real M2.[4] The nominal series are the nominal quantity of money measured by M2 and the price level measured by the Consumer Price Index (CPI). The data are from the Federal Reserve Bank of St. Louis with three exceptions. Two series, orders for durable goods and orders for industrial

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[4] The usefulness of the monetary base and M1 for the whole period are compromised by the introduction of sweep accounts in the 1990s.

machinery and equipment, are from the Census Bureau of the U.S. Government. Total employment is from the Bureau of Labor Statistics.

The data used are for January 1953 through June 1997 with exceptions where data are not available for this entire period. The series other than the financial series are seasonally adjusted at the source, with a few exceptions which we seasonally adjust using monthly dummy variables. Our intent is to focus on nonlinearities at business-cycle not seasonal frequencies; seasonally adjusted data are likely to simplify the analysis without going to the extreme of using annual-average data.[5]

We estimate autoregressions for all series to remove linear serial dependence. Each autoregression includes a constant for time and the number of lagged dependent variables indicated by the corrected Akaike Information Criterion (Hurvich and Tsai 1989.) All of these regressions are estimated in the levels. If the regressions always include at least one lag, all test statistics on additional estimated coefficients and functions of them have the usual asymptotic distributions (Sims, Stock and Watson 1990). We use the minimum value of the corrected Akaike Information Criterion (AIC_C) to determine the number of lags to include in the regression, with the search ranging from values over 40 to one for all series. In no case is the number of lags equal to the maximum examined. The first column of Table 1 shows the number of lags indicated by the AIC_C statistics. We use the Ljung-Box test (1978) to examine whether the residuals of the final equation are serially correlated. This test uniformly indicates no remaining serial correlation in the residuals at either 12 or 24 lags.

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[5] This of course is consistent with seasonal cycles being informative about other issues.

Augmented Dickey-Fuller tests for unit roots on each series can be informative about the series. Each series is tested for one and for two unit roots. The second column of Table 1 reports the results of the tests for one unit root. These statistics are based on the autoregressions with the best lag length according to the AIC_C. The column includes the sum of the coefficients of lagged levels minus one as well as the t-ratio beneath that coefficient. With our series length, a value of the t-ratio of about -3.42 or less is inconsistent with the null hypothesis at the 5 percent significance level (Fuller 1996, p. 642). The table does not report augmented Dickey-Fuller tests for two unit roots. However, the test results are consistent with two unit roots in only two cases, the real S&P 500 and the Consumer Price Index. While the results are consistent with unit roots in virtually all of the series, some of them are marginally so. Conditioning our analysis on a choice of unit roots versus near-unit roots is unnecessary for most of our analysis (Sims, Stock and Watson 1990) and we do not do so.

Current practice also suggests examining the series for ARCH. ARCH is a commonly used representation that implies nonlinearity in the sense that the innovations in the Wold representation are not independent but does not generate predictability beyond that of a linear autoregression. ARCH can be detected by the TR^2 test, a test based on a regression of squared residuals on lagged squared residuals (Engle 1982.) The product of the number of observations in the regression, T, and the R^2 has a chi-square distribution with degrees of freedom equal to the number of lags. We use only the first lag of the squared residuals, which proves to be adequate for our purposes. The third column of Table 1 presents the values of R^2 for these regressions with the p-value of the chi-square statistic underneath. These statistics provide evidence of ARCH in all of the series with the exceptions of the 10-year Treasury bond rate and the real and nominal quantities of money. This evidence of ARCH can reflect serial correlation of the volatility of the errors or other forms of nonlinearity, an issue to which we return later in the paper.

Because ARCH is consistent with linearity of the expected value conditional on past values and these test results are consistent with ARCH, we conduct our other tests on residuals from autoregressions and residuals from autoregressions with ARCH error terms. It is not feasible to estimate models with many terms in the ARCH representation. In part to obtain parsimonious representations, generalized ARCH (GARCH) models of the errors are common, with the representation

$$\begin{aligned} & x_t = \alpha + \gamma t + \sum_{i=1}^{k} \beta_i x_{t-i} + \varepsilon_t \\ (4) \qquad & \varepsilon_t | \Psi_{\mathrm{t-1}} \sim \mathrm{N}(0, \sigma_{\mathrm{t}}^2) \\ & \sigma_t^2 = \omega + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \phi_j \sigma_{t-j}^2 \end{aligned}$$

The innovations to the autoregression with GARCH have the same general properties as do innovations to a linear autoregression. The conditional normality in equation (4) implies that the third moment is zero, $E_{t-1}\varepsilon_t^3 = 0$ and $E\varepsilon_t^3 = E[E_{t-1}\varepsilon_t^3] = 0$, as are all higher odd moments. In addition, the third moment of first differences of the unadjusted errors is zero as well.

We use log-likelihood tests to select the appropriate GARCH(p,q) models from among GARCH(0,0), GARCH(0,1), GARCH(3,1) and GARCH(1,3) models. Attempts to estimate more complicated GARCH models invariably suggested overfitting. The last column of Table 1 shows the models estimated for each series. All of the models indicated by these tests are GARCH(1,3) or GARCH(3,1).

III. **Tests on the Filtered Series**

This section presents the results of various explicit tests for nonlinearity of the observed distribution of the autoregressions' residuals.

Asymmetry

Table 2 summarizes aspects of the distribution of each autoregression's residuals and related tests on the residuals. The first two columns characterize the first and second moments of the errors. While the residuals have a mean of zero, the median is not required to be zero and often is not. The second column presents the asymptotic standard deviation of the median of the series (Stuart and Ord 1991, p. 613), suggestive of the importance of the difference between the mean and median. The p-value in column two is the p-value for a test that the median equals the mean, a conceptually simple test if not particularly powerful (Stuart and Ord 1991, pp. 949-50.) For none of the series is a p-value as small as 10 percent.

The third column of Table 2 presents the skewness coefficient, a summary measure of skewness, and the p-value for testing the hypothesis that skewness is zero. The skewness coefficient is

$$u = m_3 \,/\, m_2^{3/2}, \tag{5}$$

where m_2 and m_3 are the second and third sample moments about the mean. If a distribution is symmetric, then m_3 is zero in the population, although m_3 equal to zero does not imply a symmetric distribution. A positive value of m_3 is consistent with a larger spread of values above the median than values below the median; a negative value of m_3 is consistent with a larger spread of values below the median than values above the median. If the third moment is zero, moments for the distribution are finite up to the sixth and the median is known under the null hypothesis, as it is for regression residuals, then the statistic u has an asymptotic normal distribution with variance $(\mu_6 - 6\sigma^2\mu_4 + 9\sigma^6)/T\mu_2^3$, where μ_n is the *n*th moment and σ is the standard deviation. These results provide little evidence of skewness in the series. The unemployment rate and the real S&P 500 index are the only variables with p-values less than ten percent.

An alternative test for symmetry does not require finite moments up to the sixth. Instead, it relies on two nonparametric statistics that only assume continuity of the distribution. The sign test is a test whether the median equals the mean using the statistic

$$S = T_p - ET_p, \tag{6}$$

where T_p is the number of observations with values greater than the mean, T is the total number of observations and $ET_p = T/2.$ If a series' median exceeds its mean, the value of (6) is positive and the p-value indicates the probability of a deviation that large if the population has no deviation. The p-value of the statistic (6) is computed using the exact binomial distribution under the null hypothesis that the median of the autoregression's errors is zero.

If the median equals the mean, the Wilcoxon signed rank test determines whether a series is drawn from a population that is symmetric around the mean. This statistic uses the magnitude of the deviation from the median measured by the rank of the absolute values. The test statistic is the sum of the ranks of the absolute values multiplied by an indicator of whether the observation is above or below the mean. This can be written

$$SR = \sum_{t=1}^{T} I_t r_t, \quad \text{where } I_t = \begin{cases} 1 \text{ if } \varepsilon_t > 0 \\ -1 \text{ if } \varepsilon_t \le 0 \end{cases} \tag{7}$$

where r_t is the rank of the absolute value of observation *t* and, for simplicity, we suppress the possibility of ties. The distribution of *SR* for samples greater than twenty is approximated by a *t* distribution.

The fourth and fifth columns of Table 2 report the results of these two sign tests. The sign test of whether the median equals the mean yields no p-value of 5 percent or less and p-values less than 10 percent only for the unemployment rate and the real S&P 500 index. The hypothesis that the distribution is symmetric around the mean is consistent with the data for all of these series.

The sixth column of Table 2 shows the result of the triples test (Randles *et al.* 1980; Verbrugge 1997; Hollander and Wolfe 1999, pp. 87-94.) This test is based on the function

$$f^*(x_i, x_j, x_k) = \operatorname{sgn}(x_i + x_j - 2x_k) + \operatorname{sgn}(x_i + x_k - 2x_j) + \operatorname{sgn}(x_j + x_k - 2x_i), \tag{8}$$

where $\operatorname{sgn}(y) = \{-1,0,1\}$ as $\{y < 0, y = 0, y > 0\}$. A triple is skewed to the right if the function $f^*(\ldots)$ equals one, skewed to the left if the function equals minus one and not skewed if the function equals zero. The triples test is based on the sum of all of the values of (8) for all triples in the series. This sum relative to its standard deviation is asymptotically normally distributed. This test contributes little evidence of asymmetry in the residuals for the series other than the unemployment rate and the real S&P 500 index.

A more precise and restrictive form of linearity is implied by normally distributed innovations in linear autoregressions. We use the Shapiro-Wilk test for normality (Shapiro and Wilk 1965.) This test is based on a comparison of an estimator of the variance based on the spread of sorted errors relative to the usual estimator of the error variance. The test statistic is

$$(9) \qquad W = [\sum_{i=1}^{n} a_{iT} (\varepsilon^{s}_{(T-i+1)} - \varepsilon^{s}_{i})]^{2} \;/\; \sum_{t=1}^{T} \varepsilon_{t}^{\;2}$$

where $\varepsilon^s_1 \le \varepsilon^s_2 \le \ldots \le \varepsilon^s_T$ is the sequence of ordered residuals, $n = T / 2$ if the number of observations is even and $n = (T-1) / 2$ if the number of observations is odd, and the values of the constants a_{iT} are estimated using the scheme discovered by Royston (1982.) These test results reported in the eighth column of Table 2 are not consistent with a normal distribution for most of the series.

In summary, these tests with residuals from autoregressions in Table 2 generally are consistent with a symmetric distribution of the errors but generally are inconsistent with a normal distribution. This outcome may reflect nothing more than consistency with the earlier test results in Table 1 suggesting ARCH.

Table 3 presents summary statistics and test statistics for residuals adjusted for autoregressive conditional heteroskedasticity indicated by the estimated GARCH models. A non-zero mean of these residuals is possible because the equations are not estimated by ordinary least squares. The mixed outcome of the results is apparent when comparing them to their counterparts in Table 2. More evidence of a skewed distribution is provided by the test for zero skewness in the GARCH residuals than was the case for the least-squares estimates: the p-values are less than 5 percent for the unemployment rate and the real S&P 500 index, and the p-values are between 5 and 10 percent for manufacturing employment and all of the interest rates other than the real Treasury-bill rate. On the other hand, the nonparametric tests of residuals for the GARCH estimates are similar to those for the OLS estimates. The ordinary sign test is consistent with equality of the mean and median for all series at any usual significance level and the Wilcoxon signed rank test is consistent with symmetry around the median.

The triples test provides some evidence of asymmetry for the unemployment rate, the federal funds rate and the real S&P 500 index at the 5 percent significance level and for real retail sales, the commercial paper rate and the Treasury bill rate at the 10 percent significance level. The Shapiro-Wilk test statistics for normality are inconsistent with a normal distribution at p-values less than 5 percent for real disposable income, manufacturing employment, the federal funds rate, and the real S&P 500 index. Inconsistencies with a normal distribution at a 10 percent significance level occur for industrial production, orders for industrial machinery, and the Treasury bill rate.

Overall, we interpret the results as consistent with symmetric, non-normal distributions, although the results from the triples test provide some evidence inconsistent with that interpretation for individual series, especially the unemployment rate, the federal funds rate and the real S&P 500 index.

Time Reversibility

We also examine these series for evidence of time reversibility. Tests for symmetry of *first differences* of the residuals are tests for time reversibility (Chen, Chou and Kuan 2000.) We use the tests of symmetry above to test for time reversibility. For brevity, these test results are omitted: virtually all of the series appear to be consistent with symmetric changes in the residuals. Among the unadjusted and the adjusted series, residuals for the real S&P 500 index furnish the only indications of departures from symmetry in the first differences of contemporaneous residuals. This conclusion is difficult to reconcile with conflicting graphs of series such as the unemployment rate, but these are tests based on only contemporaneous asymmetries.

A more comprehensive test for time reversibility, the TR test, relies on serial third-order moments (Ramsey and Rothman 1993, 1996). Under the null hypothesis, a zero-mean time-reversible series has

$$(10) \qquad Ex_t^2 x_{t-j} = Ex_t x_{t-j}^2, \quad j > 0 \qquad ,$$

the insight exploited by Ramsey and Rothman's test. Their test also can be used to determine whether any asymmetry is due to nonlinearity or to non-normal errors. If the series used in the test is serially uncorrelated, the test statistic

$$(11) \qquad \frac{[\mu_{2,1}(j) - \mu_{1,2}(j)]/(T-j)}{[(2\mu_4 - \mu_3^2)/(T-j) - 2\mu_2^3(T-2j)/(T-j)^2]^{1/2}}$$

has an asymptotic normal distribution, where $\mu_{h,i}(j) = Ex_t^h x_{t-j}^i$. If the series is serially correlated, the asymptotic and finite-sample distributions are not known.

Applying the TR test to both the residuals from estimated equations as well as the unfiltered series makes it possible to resolve whether any time irreversibility in the original series is due solely to non-normally distributed innovations. If the unfiltered series is inconsistent with time reversibility, this could be due to either a nonlinear difference equation generating the series or to non-normally distributed innovations. The residuals from the estimated autoregressions are i.i.d. for a correctly specified linear equation; all of the estimated serial third-order moments in (10) are zero and therefore equal to each other. As a result, the test statistic (11) is inconsistent with time reversibility for the series but not the residuals if a linear autoregression is an adequate representation of the difference equation generating the series. In general, the test will be inconsistent with time reversibility for both the series and the residuals if the difference equation is nonlinear. Table 4 summarizes this discussion.

Following Ramsey and Rothman (1996), we examine the maximum value of a set of N test statistics for several lags rather than examine numerous test statistics for time reversibility for each series. The asymptotic normal distribution of the test statistic for serially uncorrelated series implies that the asymptotic distribution for the maximum value of N statistics is known. Our tests are applied to unfiltered data for which the asymptotic distribution is not known and to the residuals for which the asymptotic distribution is known. Because it is necessary to conduct a Monte Carlo analysis for the unfiltered series, it is low cost to perform one for the residuals as well to get more comparable finite-sample distributions. Monte Carlo simulations of the estimated equations with normally distributed innovations generate an estimate of the finite-sample distribution of the test statistic for the unfiltered series under this null hypothesis. If the test statistic (11) for the unfiltered series is large in absolute value relative to the values generated by the simulations, then the series is inconsistent with a linear equation, normally distributed innovations or both.[6] Monte Carlo simulations for normally distributed innovations generate an estimate of the finite-sample distribution of the test statistics for the innovations under the null hypothesis. If this test statistic (11) is large in absolute value relative to the values generated by the simulations for the innovations and the test statistic for the unfiltered series is large as well, then these combined results indicate that a nonlinear difference equation underlies the behavior of the series.

Table 5 presents the test results for time reversibility applied to the original series and to the residuals from the linear autoregressions and the autoregressions with GARCH. This test must be applied to the original data not just to the data filtered by autoregressions, and these unfiltered data must be stationary for the test statistics to have known asymptotic properties. Hence, we must choose the detrended or differenced series. We use the differenced series. The unit root tests are consistent with this choice and the detrended series persist above and below trend for ten years or more, indicating that a linear trend is not adequately stabilizing the series to a stationary series.

The results for the autoregressions estimated by OLS generally suggest a nonlinear difference equation. The relevant p-values are in the first two columns of Table 5. The statistics for the original series are inconsistent with time reversibility at the 5 percent significance level with the exceptions of the unemployment rate and orders for durable goods. This result is

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[6] Ramsey and Rothman (1996) re-estimate the parameters of the process generating the data in the simulations. This refinement deals with estimation error, which seems to us to be a third-order problem.

consistent with the hypothesis that the underlying difference equation is nonlinear or the innovations are asymmetric. The statistics for the residuals generally have p-values less than 5 percent, with the exceptions of the inventory-sales ratio, orders for durable goods, the real Treasury-bill rate, the real money stock and the CPI. Overall, the results suggest that nonlinear difference equations underlie the data.

The second two columns of Table 5 show p-values of the TR test for the series with GARCH parameters estimated.[7] The inconsistency of the original series with a linear difference equation and normally distributed innovations is much less general. For example, only four of the residual series are inconsistent with the null hypothesis of i.i.d. normality at a one percent significance level. Overall, these results with the GARCH parameters estimated suggest that the non-financial series may still have an additional nonlinear component but the financial and monetary series generally are consistent with autoregressions with GARCH innovations that are normally distributed.

Other Tests for Nonlinearity

There are numerous tests for nonlinearity in addition to those above. Table 6 and 7 present p-values of the suite of tests in Ashley and Patterson (1998.) The tests are the bispectral test (Hinich 1982), the BDS test (Brock, Dechert and Scheinkman 1996), the bicovariance test (Hinich 1996) and Tsay's test for a threshold autoregression (Tsay 1989.) We do not repeat Ashley and Patterson's summary of the tests, other than to note that the first three tests are tests for

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[7] It might seem that the ARCH terms would not affect the test for the original series, but the estimated model is used in conjunction with normally distributed innovations to generate the p-value.

nonlinearity without a specific alternative hypothesis and Tsay's test is designed to do particularly well for the alternative of a threshold autoregression. All p-values are estimated by comparing the computed test statistics with the relative frequency for i.i.d. normal series with the number of observations in the residual series.

Table 6 provides substantial evidence of nonlinearity. The bispectral test has two parts: first a test whether the series is normal and, then conditional on the series being normal, a test whether the series is linear. This test provides little evidence of nonlinearity or even non-normality. The BDS test, however, which examines the closeness of observations in spaces of various dimensions — two and three in Table 6 — is inconsistent with linearity for all of the series. Similarly the bicovariance test is inconsistent with linearity for almost all of the series, and Tsay's TAR test also is inconsistent with linearity.

The test statistics in Table 7 based on standardized residuals from the autoregressions with GARCH provide little evidence of nonlinearity. Some of the series may be inconsistent with a linear autoregression with GARCH innovations from a normal distribution, but not many.

GARCH Models versus Nonlinearly Predictable

These tests suggest that there may well be some nonlinearity in these data, although the tests strongly suggest that much of the nonlinearity and non-normality is associated with ARCH in the residuals. This particular form of nonlinearity is interesting because it can be consistent with linear functions for the conditional expected values.

Still, this empirical result suggesting ARCH and little else in terms of asymmetry is odd. The distinction between movements in recessions and those in expansions is conspicuous in the graphs of the unemployment rate and industrial production in Figure 1. Figure 2 suggests that this apparent nonlinearity is no longer so obvious after estimating a time-varying conditional variance. Figure 2 displays the residuals, conditional standard deviation and normalized residuals for the unemployment rate. The apparent large changes in the unemployment rate in recessions become muted because the first large unpredicted change increases the conditional standard deviation, which then deflates the succeeding large unpredicted changes. In sum, the apparently large sequential changes are transformed into one relatively large change followed by sequential changes.

Are nonlinearities associated with autoregressive conditional heteroskedasticity the only ones in these data? We examine whether the occurrence of high conditional standard deviations is associated with recessions, as Figure 2 might suggest. While such an association does not imply that the series have any predictability beyond GARCH errors, it can provide evidence concerning the generality of the pattern in Figure 2.

Table 8 reports statistics for chi-square tests of association between high conditional standard deviations and recessions. We summarize the tests by presenting: 1. the expected proportion of observations in recessions that have high conditional standard deviations if such observations have the same probability in recessions as in expansions; and 2. the actual proportion of observations in recessions that have high conditional standard deviations. These proportions are presented in the first, third and fifth columns of Table 8 for different durations of episodes with high standard deviations. We define a period with a high conditional standard deviation as a period with a conditional standard deviation that exceeds the sample-average conditional standard deviation. We define episodes with a high conditional standard deviation as being episodes in which respectively: 1. one period has a high standard deviation (one month duration); 2. this period and two surrounding periods have high standard deviations (three months duration); and 3. this period and five consecutive surrounding periods have high standard deviations (six months duration).

Episodes of high conditional standard deviations are more common in recessions than otherwise for all but one of the series. High conditional standard deviations for real disposable income never are observed in recessions, which is a complete reversal of the computed frequency for other series. For the other series, the greater frequency of high conditional standard deviations in recessions is statistically significant at the 5 percent significance level with only a few exceptions. The exceptions are: total employment and the CPI with one-period durations of high conditional standard deviation; machinery orders, total employment and manufacturing employment with three-period durations of high conditional standard deviations; and total employment with six-period durations of high conditional standard deviations.

Does this association between episodes of high conditional standard deviations and recessions indicate predictable changes in the series in addition to what is predicted by a linear autoregression? We approach this issue in a simply way. The conditional standard deviation itself is predictable by the ARCH estimates.[8] We separate periods with high and low predicted conditional standard deviations. We then examine whether different autoregressions better

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[8] This is advantageous relative to the alternative of using recessions to separate the periods because we have no reason to claim that recessions are predictable.

characterize these separate periods than a single one. The last column of Table 8 presents statistics summarizing the results of estimating different regressions for episodes with a high standard deviation and episodes with a low standard deviation. The statistics are the p-values from F-ratios testing whether the estimated coefficients are different in episodes with low and high standard-deviations. We present only p-values for episodes with one-month and six-month durations. The results for three-month durations are similar.

The regressions are quite consistent with the proposition that the series have a component predictable from their own history in addition to that predictable by a linear autoregression. If an episode is defined to be one period of high standard deviation -- clearly predictable by the estimated equation -- 9 of the 18 series are inconsistent with a single linear regression at the 5 percent significance level. As episodes are defined to include more consecutive periods of one-step-ahead forecasts of high standard deviations, the evidence of predictability increases. Defining an episode of high standard deviation as six consecutive months of high standard deviation, only the inventory-sales ratio and the two orders series are consistent with the same linear regression being sufficient to characterize the data in high and low standard deviation periods. We conclude that there is substantial evidence of GARCH models' inability to adequately characterize the nonlinearity and that the series are nonlinearly predicable.

Asymmetric Adjustment Costs

The preceding evidence is suggestive of asymmetric adjustment costs in contractions and expansions. A preliminary characterization of the differences in the series' behavior in contractions and expansions would provide more direct evidence. It is well beyond the scope of this paper to provide a joint dynamic nonlinear model of the economy. Fortunately, it is not necessary to estimate such a model in order to provide evidence that work on such a model is likely to be informative.

If asymmetric adjustment costs are important, then deviations of output from capacity in contractions are likely to be characterized by different regressions than deviations of output in expansions. Threshold autoregressions are a simple way to characterize different behavior in contractions and expansions. Threshold autoregressions are piecewise linear equations. A three-region threshold autoregression can be written

$$(12) \qquad x_t = \begin{cases} \alpha_1 + \beta_1(L)x_{t-1} + \varepsilon_{1,t} & \text{if } z_t \geq r_1 \\ \alpha_2 + \beta_2(L)x_{t-1} + \varepsilon_{2,t} & \text{if } \mathrm{r}_1 > z_t > r_2 \\ \alpha_3 + \beta_3(L)x_{t-1} + \varepsilon_{3,t} & \text{if } \mathrm{r}_2 \geq z_t \end{cases}$$

where the applicable linear regression is determined by the value of a variable z_t compared to the parameters r_i, i=1,2 for the three regions. In many macroeconomic applications such as Potter (1994), the variable z_t is a lagged value of the left-hand-side series, x_{t-d} where d is the lag used. The variable z_t can be functions of other variables as in studies of index arbitrage for example (Dwyer, Locke and Yu’s 1996; Tsay 1998.) In the context of our analysis, variables that reflect contractions and expansions are the pertinent ones, and the point of the analysis is to examine common behavior. This suggests choosing one variable to determine the switch points for all of the variables.

Among those included in our analysis, changes in the unemployment rate and industrial production growth are plausible candidates for switching variables. There are statistical procedures for estimating the values of the parameters determining the regions, r_i, as well as the delay of the switching variable. We pursue a simpler course. Figure 3 shows changes in the unemployment rate and the monthly growth rate of industrial production. Because the unemployment rate's reported precision is tenths of a percentage point, the number of possible values that it takes is rather limited. Reference lines drawn at plus and minus three-tenths of a percentage point suggest that these values are likely to be informative. Regions delineated by these values would pick out relatively large changes but there are enough large changes that the equations are likely to be estimable with reasonably precision. Most increases in the unemployment rate by 0.3 percent or more are during recessions and many decreases that large in magnitude or larger are shortly after the recessions. The growth rate of industrial production in the bottom part of the figure shows decreases in recessions -- it hardly could be otherwise -- but the graph suggests to us that regions based on industrial production's growth are likely to be less satisfactory.

Rather than search over possible delays between the changes in the unemployment rate, we present some preliminary evidence based on a delay of one month. There is no necessary reason to restrict the delay to one month, and indeed given that our series include leading, contemporaneous and lagging indicators of the economy, it is unlikely to be optimal. This restriction will bias our results against finding different regressions. Still, asymmetric adjustment costs are hardly only possible reason for nonlinearity. For example, Choi (1999) and Weise (1999) find nonlinearity that appears to be associated with monetary policy and Peltzman (1999) finds asymmetries in individual markets that may be reflected in the aggregate price level. Overall, we conclude that restricting the delay to one month limits the likelihood that we will find nonlinearity due to some something other than changes in the aggregate economy.

Table 9 presents F-statistics for evaluating the important of piecewise linear regressions. All of the regressions include all of the lags indicated by the AIC_C for linear regressions, which is correct under the null hypothesis of linearity although it is likely to include too many parameters under the alternative of a threshold autoregression (TAR.) The first column of Table 9 presents F-ratios, degrees of freedom and p-values for testing the hypothesis that coefficients are the same for a region based on increases in the unemployment rate and the rest of the data.

In general, the data are inconsistent with a single linear autoregression. For twelve of the nineteen series, the F-ratios in the first column are inconsistent with the null hypothesis of a linear regression at the 5 percent significance level for twelve of the nineteen series and at the 10 percent significance level for fifteen of the nineteen series. There is no particular pattern to the failures to reject the null hypothesis. We conclude that the data provide substantial support for the proposition that the behavior of macroeconomic series is different in contractions than in other times relative to the null hypothesis that all of the series are well characterized by linear autoregressions.

The second and third columns of Table 9 present test statistics related to testing whether the difference in behavior in symmetric, with periods of large decreases in the unemployment rate also characterized by different autoregressions. This hypothesis might reflect the possibility that there is a recovery period that is characterized by a different linear autoregression than the rest of expansions. The F-ratios in the second column can be used to test the null hypothesis that all coefficients are the same in all three regions in equation (12). There is some evidence inconsistent with this null hypothesis for real disposable income and the interest rates but not for the rest of the series. Given that the test statistics in the first column indicate that increases in the unemployment rate of 0.3 percentage points or more are associated with different coefficients, this suggests that decreases in the unemployment rate may be similar to changes other than these relatively large increases. The third column indicates that this hypothesis is consistent with the data for all series but real disposable income and the short-term interest rates.

Overall, we conclude that there is substantial evidence of asymmetric behavior associated with contractions. Our preliminary evidence does not support for a recovery period different than the rest of expansions, as would be suggested by Beaudry and Koop (1993.)

IV. **Conclusion**

This examination of output series, financial and monetary series, and nominal series clearly indicates that there is a common nonlinear component associated with recessions. The tests for ARCH, time reversibility and nonlinearity are consistent with nonlinearity. GARCH errors in a linear autoregression misses a predictable component of the series. Our simple estimates of threshold autoregressions based on changes in the unemployment rate indicate that a common factor associated with recessions is important.

Our evidence indicates that there is asymmetry in a number of series not ordinarily considered, including employment, interest rates, and money balances. Such asymmetries are likely if there are asymmetries in output and the series are related, as economic theory would indicate. We are inclined to think that advances beyond our preliminary estimates will require serious work on the asymmetric adjustment costs associated with recessions. As in Dwyer, Locke and Yu (1996), we find it helpful to examine the identification problem using economic theory as a loose guide for solving the problem.

REFERENCES

Ashley, Richard A., and Douglas M. Patterson. 1989. "Linear versus Nonlinear Macroeconomics: A Statistical Test." *International Economic Review* 30 (August), 685-704.

____, and ____. 1998. “Nonlinear Model Specification/Diagnostics: Insights from a Battery of Nonlinearity Tests.” Unpublished paper, Virginia Tech.

Balke, Nathan S., and Thomas B. Fomby. 1994. "Large Shocks, Small Shocks, and Economic Fluctuations: Outliers in Macroeconomic Time Series." 9 *Journal of Applied Econometrics* (April-June), 181-200.

Barnhart, Cora. 1995. *Nonlinearity and Business Cycles*. Unpublished Ph.D. dissertation, Clemson University.

Beaudry, Paul, and Gary Koop. 1993. "Do Recessions Permanently Change Output?" *Journal of Monetary Economics* 31 (April), 149-63.

Brock, William A., and W. D. Dechert, J. A. Scheinkman, and B. LeBaron. 1996. *Econometric Review*s “A Test for Independence Based on the Correlation Dimension.” 15 (August), 197-235.

________, and Chera L. Sayers. 1988. "Is the Business Cycle Characterized by Deterministic Chaos?" *Journal of Monetary Economics* 22(July), 71-90.

Chen, Yi-Ting, Ray Y. Chou and Chung-Ming Kuan. 2000. “Testing for Time Reversibility without Moment Restrictions.” Forthcoming, *Journal of Econometrics*.

Choi, Woon Gyu. 1999. “Asymmetric Monetary Effects on Interest Rates across Monetary Policy Regimes.” *Journal of Money, Credit and Banking* 31 (August), 386-416.

Cao, C. Q., and Ruey S. Tsay. 1992. "Nonlinear Time-Series Analysis of Stock Volatilities." *Journal of Applied Econometrics* 7 (December), S165-85.

DeLong, J. Bradford, and Lawrence H. Summers. 1986. "Are Business Cycles Symmetrical?" In R. J. Gordon, ed., *The American Business Cycle*. Chicago: NBER and University of Chicago Press.

Dwyer, Gerald P., Jr., Peter Locke and Wei Yu. 1996. “Index Arbitrage and Nonlinear Dynamics between the Futures and Cash S&P 500.” *Review of Financial Studies* 9 (Spring), 301-32.

Engle, Robert F. 1982. “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation.” *Econometrica* 50 (July), 987-1007.

The Economist: Guide to Global Economic Indicators 1994. New York: John Wiley & Sons, Inc.

Friedman, Milton. 1993. “The ‘Plucking Model’ of Business Fluctuations Revisited.” *Economic Inquiry* 31 (April), 171-77.

Fuller, Wayne A. 1996. *Introduction to Statistical Time Series*. Second edition. New York: John Wiley & Sons, Inc.

Gupta, M. K. 1967. “An Asymptotically Nonparametric Test of Symmetry.” *Annals of Mathematical Statistics* 38 (June): 849-66.

Hallin, Marc, Claude Lefevre and Madan L. Puri. 1988. “On Time-reversibility and the Uniqueness of Moving Average Representations for Non-Gaussian Stationary Time Series.” *Biometrika* (March), 170-1.

Hicks, John R. 1950. *A Contribution to the Theory of the Trade Cycle*. London: Oxford at the Clarendon Press.

_________. 1982. "Are There Economic Cycles?" In *Money, Interest, and Wages,* pp. 332-46. Cambridge: Harvard University Press.

Hinich, Melvin J. 1982. “Testing for Gaussianity and Linearity of a Stationary Time Series.” *Journal of Time Series Analysis* 3, 169-76.

_________. 1996. “Testing for Dependence in the Input to a Linear Time Series Model.” *Nonparametric Statistics* 6, 205-21.

_________, and Douglas M. Patterson. 1985. "Evidence of Nonlinearity in Daily Stock Returns." *Journal of Business and Economic Statistics* 3 (January), 69-77.

Hollander, Myles, and Douglas A. Wolfe. 1999. *Nonparametric Statistical Methods.* Second edition. New York: John Wiley & Sons, Inc.

Hsieh, David A. 1991. "Chaos and Nonlinear Dynamics: Application to Financial Markets." *Journal of Finance* 46 (December): 1839-77.

Hurvich, Clifford M. and Chih-Ling Tsai. 1989. “Regression and Time Series Model Selection in Small Samples.” *Biometrika* 76 (June): 297-307.

Jansen, Dennis, and Wankeum Oh. 1996. “Modeling Nonlinearity of Business Cycles: Choosing between the CDR and STAR Models.” Unpublished paper, Texas A&M University.

Kelly, F. P. 1979. *Reversibility and Stochastic Networks*. Chichester, England: John Wiley & Sons Ltd.

Ljung, G. M., and G. E. P. Box. 1978. "On a Measure of Lack of Fit in Time Series Models." 65 *Biometrika* (August), 297-303.

Marsh, Terry A. 1985. "On Nonlinear Serial Dependencies in Stock Returns." *Journal of Econometrics* 30 (1):289-96.

Mizrach, Bruce 1994. "Using U-Statistics to Detect Business Cycle Nonlinearities." In Willi Semmler, ed., *Business Cycles: Theory and Empirical Methods*. Boston: Kluwer Academic Press, pp. 369-88.

Montgomery, Alan L., Victor Zarnowitz, Ruey S. Tsay and George C. Tiao. 1998. “Forecasting the U.S. Unemployment Rate.” *Journal of the American Statistical Association* 93 (June), pp. 478-93.

National Bureau of Economic Research. “US Business Cycle Expansions and Contractions.” http://www.nber.org/cycles.html on 12/17/98.

Peltzman, Sam. 1999. “Prices Rise Faster than They Fall.” Forthcoming, *Journal of Political Economy*.

Potter, Simon M. 1994. "Asymmetric Economic Propogation [sic] Mechanisms." In Willi Semmler, ed., *Business Cycles: Theory and Empirical Methods*. Boston: Kluwer Academic Press, pp. 313-30.

Priestley, M. B. 1988. *Non-linear and Non-stationary Time Series Analysis*. San Diego: Academic Press Inc.

Ramsey, James B., and Philip Rothman. 1993. "Time Irreversibility and Business Cycle Asymmetry." Unpublished manuscript, Department of Economics, New York University and East Carolina University.

________, and _________. 1996. “Time Irreversibility and Business Cycle Asymmetry.” *Journal of Money, Credit and Banking* 28 (February 1996): 1-21.

Randles, Ronald H., Michael A. Fligner, George E. Policello II and Douglas A. Wolfe. 1980. “An Asymptotically Distribution-Free Test for Symmetry Versus Asymmetry.” *Journal of the American Statistical Association* 75 (March), 168-72.

Rothman, P. 1992. "The Comparative Power of the TR Test Against Simple Threshold Models." *Journal of Applied Econometrics* 7 (December), S187-S195.

Rothman, P. 1994. "The Time Irreversibility Test with Application to Financial Data." In Willi Semmler, ed., *Business Cycles: Theory and Empirical Methods*. Boston: Kluwer Academic Press.

Royston, J. P. 1982. “An Extension of Shapiro and Wilk’s W Test to Large Samples.” *Applied Statistics* 31 (2): 115-24.

Shapiro, S. S., and M. B. Wilk. 1965. “An Analysis of Variance Test for Normality (Complete Samples).” *Biometrika* 52 (3 and 4): 591-611.

Sims, Christopher A., James H. Stock and Mark W. Watson. 1990. "Inference in Linear Time Series Models with Some Unit Roots." *Econometrica* 58 (January), 113-44.

Sterman, J. 1989. "Deterministic Chaos in An Experimental Economic System." *Journal of Economic Behavior and Organization* 12:1-28.

Stuart, Alan, and J. Keith Ord. 1991. *Kendall’s Advanced Theory of Statistics*. Two volumes. New York: Oxford University Press.

Tong, Howell. 1990. *Nonlinear Time-Series: A Dynamical System Approach*. New York: Oxford University Press.

Tsay, Ruey S. 1986. "Nonlinearity Tests for Time Series." *Biometrika* 73 (August), 461-66.

Tsay, Ruey S. 1989. "Testing and Modeling Threshold Autoregressive Processes." *Journal of the American Statistical Association* 84 (March), 231-40.

________. 1998. “Testing and Modeling Multivariate Threshold Models.” *Journal of the American Statistical Association* 93 (September), 1188-1202.

Verbrugge, Randal. 1997. “Investigating Cyclical Asymmetries.” *Studies in Nonlinear Dynamics and Econometrics* 2 (April), 15-22.

Weise, Charles L. 1999. "The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregression Approach." *Journal of Money, Credit and Banking* 31 (February), pp. 85-108.

Weiss, Gideon. 1975. "Time-Reversibility of Linear Stochastic Processes." *Journal of Applied Probability* 12 (December), 831-36.

Whittle, P. 1975. "Reversibility and Acyclicity." In *Perspectives in Probability and Statistics*. Edited by J. Gani. London: Applied Probability Trust.

Table 1. SUMMARY OF ESTIMATED REGRESSIONS

VARIABLE	LAGS	ONE UNIT ROOT COEFFICIENT T-RATIO	R^2 SQUARED RESIDUALS P-VALUE	PARAMETERIZATION GARCH (p,q)[a]
Industrial Production	27	-0.006 -1.35	0.043 0.001	(1,3)
Real Disposable Income	3	-0.007 -1.10	0.086 $<10^{-3}$	(3,1)
Real Retail Sales	4	-0.021 -2.20	0.060 $<10^{-3}$	(3,1)
Total Employment	8	-0.021 -3.27	0.041 0.001	(3,1)
Manufacturing Employment	3	-0.009 -2.52	0.063 $<10^{-3}$	(3,1)
Unemployment Rate	25	-0.017 -2.30	0.023 0.071	(3,1)
Inventory-sales Ratio	25	-0.050 -2.70	0.011 0.0494	(3,1)
Total Capacity Utilization	4	-0.031 -3.41	0.070 $<10^{-3}$	(1,3)
Orders for Durable Goods	7	-0.028 -2.24	0.016 0.266	(3,1)
Orders for Industrial Machinery	7	-0.029 -2.35	0.034 0.014	(3,1)
Federal Funds Rate	17	-0.009 -2.92	0.190 $<10^{-3}$	(3,1)
Commercial Paper Rate	14	-0.027 -2.81	0.037 0.004	(3,1)
90-Day Treasury-Bill Rate	22	-0.019 -2.08	0.018 0.002	(3,1)
Ten-Year Treasury Bond Rate	16	-0.007 -1.34	0.016 0.210	(3,1)
Real Treasury-Bill Rate	25	-0.080 -3.31	0.028 0.022	(3,1)
Real S&P 500 Index	2	-0.002 -0.40	0.024 0.049	(3,1)
Real Money Stock — M2	10	-0.005 -2.74	0.022 0.121	(1,3)
Money Stock — M2	11	0.001 0.59	0.012 0.472	
Consumer Price Index	33	-0.001 -1.96	0.028 $<10^{-3}$	(3,1)

[a] The GARCH parameterization of the residual is based on $\sigma_i^2 = \omega + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \varphi_j \sigma_{t-j}^2$.

Table 2. CHARACTERISTICS OF DISTRIBUTIONS OF RESIDUALS

VARIABLE	MEAN (MEDIAN)	STANDARD DEVIATION (P-VALUE)	SKEWNESS COEFFICIENT (P-VALUE)	SIGN TEST (P-VALUE)	WILCOXON SIGNED RANK TEST (P-VALUE)	TRIPLES TEST (P-VALUE)	SHAPIRO-WILK (P-VALUE)
Industrial Production	0.00	0.001	0.995	11.5	304.5	-1.115	0.956
	$3.33\cdot10^{-4}$	0.535	0.160	0.341	0.932	0.265	$<10^{-3}$
Real Disposable Income	0.00	0.001	0.619	2.5	-981.0	0.072	0.877
	$7.2\cdot10^{-5}$	0.887	0.268	0.852	0.730	0.942	$<10^{-3}$
Real Retail Sales	0.00	0.001	1.182	15.5	2082.5	-1.653	0.984
	$4.89\cdot10^{-4}$	0.537	0.119	0.194	0.559	0.098	0.336
Total Employment	0.00	$<10^{-3}$	0.563	0.5	811.5	-0.780	0.980
	$2.68\cdot10^{-6}$	0.990	0.287	1.000	0.820	0.435	0.033
Manufacturing Employment	0.00	$<10^{-3}$	0.120	-1.5	1760.5	-0.876	0.824
	$-7.48\cdot10^{-6}$	0.980	0.452	0.931	0.621	0.381	$<10^{-3}$
Unemployment Rate	0.00	0.012	2.475	-21.5	-3171.5	2.591	0.979
	-0.0163	0.163	0.007	0.069	0.373	0.010	0.032
Inventory-sales Ratio	0.00	0.001	1.191	-10.5	-1024.5	0.261	0.985
	$-4.8\cdot10^{-5}$	0.668	0.117	0.386	0.774	0.794	0.449
Total Capacity Utilization	0.00	0.046	0.107	5.0	117.5	-0.301	0.981
	0.025	0.588	0.457	0.636	0.953	0.764	0.216
Orders for Durable Goods	0.00	0.002	0.431	7.0	431.5	-0.725	0.987
	0.001	0.505	0.333	0.547	0.882	0.468	0.692
Orders for Industrial Machinery	0.00	0.003	0.723	-9.0	-1305.5	1.009	0.980
	-0.001	0.625	0.235	0.431	0.654	0.313	0.074
Federal Funds Rate	0.00	$3.47\cdot10^{-4}$	0.749	-8.5	-1744.0	1.301	0.820
	$-9\cdot10^{-5}$	0.796	0.227	0.474	0.589	0.193	$<10^{-3}$
Commercial Paper Rate	0.00	$3.46\cdot10^{-4}$	1.155	-18.5	-1203.0	1.554	0.838
	$-2.2\cdot10^{-4}$	0.525	0.124	0.115	0.728	0.120	$<10^{-3}$
90-Day Treasury-Bill Rate	0.00	$2.91\cdot10^{-4}$	0.299	-8.5	-1268.5	0.735	0.865
	$-1.2\cdot10^{-4}$	0.680	0.382	0.488	0.722	0.462	$<10^{-3}$
Ten-Year Treasury Bond Rate	0.00	$1.67\cdot10^{-4}$	1.100	-13.5	-1906.0	0.688	0.960
	$-1.1\cdot10^{-4}$	0.510	0.136	0.252	0.573	0.491	$<10^{-3}$
Real Treasury-Bill Rate	0.00	0.001	0.811	-1.5	505.5	0.252	0.979
	$-1\cdot10^{-4}$	0.876	0.209	0.931	0.887	0.801	0.024
Real S&P 500 Index	0.00	0.002	2.657	21.5	3780.0	-3.140	0.969
	$2.19\cdot10^{-3}$	0.296	0.004	0.068	0.286	0.002	$<10^{-3}$
Real Money Stock — M2	0.00	$2.24\cdot10^{-4}$	0.290	4.5	441.5	-0.378	0.992
	$6.1\cdot10^{-5}$	0.785	0.386	0.707	0.874	0.706	0.981
Money Stock — M2	0.00	$1.56\cdot10^{-4}$	0.890	-7.5	-988.0	0.841	0.964
	$-6\cdot10^{-5}$	0.701	0.187	0.510	0.722	0.400	$<10^{-3}$
Consumer Price Index	0.00	$1.13\cdot10^{-4}$	1.077	-13.5	-1501.5	1.149	0.984
	$-1.3\cdot10^{-4}$	0.322	0.141	0.260	0.673	0.251	0.364

Table 3. CHARACTERISTICS OF DISTRIBUTIONS OF GARCH-ADJUSTED RESIDUALS

Variable	Mean (Median)	Standard Deviation (p-value)	Skewness Coefficient (p-value)	Sign Test (p-value)	Wilcoxon Signed Rank Test (p-value)	Triples Test (p-value)	Shapiro-Wilk (p-value)
Industrial Production	-0.072	0.064	0.654	-7.5	-5091.5	-1.012	0.980
	-0.023	0.722	0.257	0.544	0.153	0.312	0.053
Real Disposable Income	$7.18 \cdot 10^{6}$	0.070	0.620	2.5	-981.0	0.015	0.877
	$7.2 \cdot 10^{-5}$	0.888	0.268	0.852	0.730	0.988	$<10^{-3}$
Real Retail Sales	$-1.37 \cdot 10^{-3}$	0.065	0.818	10.5	1634.5	-1.675	0.988
	0.042	0.511	0.207	0.386	0.646	0.094	0.793
Total Employment	-0.028	0.065	0.817	-12.5	-2100.5	-0.326	0.989
	.049	0.452	0.207	0.299	0.555	0.745	0.895
Manufacturing Employment	0.012	0.065	1.515	11.5	5019.5	-0.661	0.922
	.045	0.489	0.065	0.341	0.158	0.509	$<10^{-3}$
Unemployment Rate	0.057	0.064	1.786	1.5	2535.5	2.099	0.987
	$4.57 \cdot 10^{7}$	1.000	0.037	0.931	0.477	0.036	0.768
Inventory-sales Ratio	$4.57 \cdot 10^{7}$	0.065	1.191	-10.5	-1024.5	0.261	0.985
	-0.028	0.667	0.117	0.386	0.774	0.794	0.449
Total Capacity Utilization	$-3 \cdot 10^{-5}$	0.078	0.106	5.0	115.5	-0.300	0.981
	0.042	0.588	0.458	0.636	0.954	0.764	0.216
Orders for Durable Goods	-0.019	0.069	0.360	4.0	-845.5	-0.478	0.989
	0.009	0.898	0.359	0.746	0.772	0.633	0.906
Orders for Industrial Machinery	$3.3 \cdot 10^{-5}$	0.069	0.726	-9.0	-1310.5	1.015	0.980
	-0.034	0.625	0.234	0.431	0.653	0.310	0.074
Federal Funds Rate	0.077	0.067	1.385	0.5	3426.0	2.319	0.973
	$2.84 \cdot 10^{-3}$	0.966	0.083	1.000	0.288	0.020	$<10^{-3}$
Commercial Paper Rate	$1.61 \cdot 10^{-4}$	$3.82 \cdot 10^{-4}$	0.599	-5.5	1236.0	1.689	0.984
	$2.3 \cdot 10^{-5}$	0.952	0.055	0.662	0.721	0.091	0.364
90-Day Treasury-Bill Rate	0.018	0.065	1.511	-10.5	-320.5	1.861	0.980
	-0.043	0.508	0.065	0.386	0.928	0.063	0.065
Ten-Year Treasury Bond Rate	0.060	0.066	1.490	7.5	3475.0	1.019	0.987
	0.037	0.569	0.068	0.537	0.304	0.308	0.722
Real Treasury-Bill Rate	0.03	0.065	1.080	14.5	3784.5	-1.117	0.990
	0.047	0.473	0.140	0.225	0.288	0.264	0.892
Real S&P 500 Index	-0.019	0.065	2.500	14.5	2038.0	-2.972	0.975
	0.052	0.424	0.006	0.224	0.565	0.003	0.001
Real Money Stock — M2	0.004	0.070	0.360	5.5	814.5	-0.422	0.992
	0.023	0.749	0.361	0.639	0.771	0.673	0.985
Money Stock — M2	NA	NA	NA	NA	NA	NA	NA
Consumer Price Index	$-8.97 \cdot 10^{6}$	0.065	1.077	-13.5	-1504.5	1.141	0.984
	-0.062	0.340	0.141	0.260	0.673	0.254	0.364

Table 4

THE IMPLICATIONS OF THE TR TEST ON THE SERIES AND THE RESIDUALS

	SERIES TIME REVERSIBLE	SERIES NOT TIME REVERSIBLE
Residuals Time Reversible	Linear equation Symmetric residuals	Linear equation Asymmetric residuals
Residuals Not Time Reversible	Cannot happen in population	Nonlinear equation Symmetric or asymmetric residuals

Table 5. TIME REVERSIBILITY TEST RESULTS

	OLS		**GARCH**	
VARIABLE	SERIES	RESIDUAL	SERIES	RESIDUAL
	Absolute Max P-Value Portmanteau P-Value			
Industrial Production	$<10^{-3}$	$<10^{-3}$	0.041	0.046
Real Disposable Income	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$
Real Retail Sales	$<10^{-3}$	$<10^{-3}$	0.004	0.011
Total Employment	$<10^{-3}$	$<10^{-3}$	0.024	0.009
Manufacturing Employment	$<10^{-3}$	$<10^{-3}$	0.094	0.283
Unemployment Rate	0.053	0.003	0.256	0.056
Inventory-sales Ratio	$<10^{-3}$	0.089	$<10^{-3}$	0.208
Total Capacity Utilization	0.003	$<10^{-3}$	0.005	$<10^{-3}$
Orders for Durable Goods	0.215	0.074	0.454	0.403
Orders for Industrial Machinery	$<10^{-3}$	0.018	$<10^{-3}$	0.013
Federal Funds Rate	$<10^{-3}$	$<10^{-3}$	0.085	0.182
Commercial Paper Rate	$<10^{-3}$	$<10^{-3}$	0.066	0.155
90-Day Treasury-Bill Rate	$<10^{-3}$	$<10^{-3}$	0.392	0.437
Ten-Year Treasury Bond Rate	$<10^{-3}$	$<10^{-3}$	0.413	0.345
Real Treasury-Bill Rate	$<10^{-3}$	0.508	0.168	0.963
Real S&P 500 Index	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$	0.003
Real Money Stock — M2	0.028	0.289	0.032	0.277
Money Stock — M2				
Consumer Price Index	0.019	0.348	0.060	0.148

Table 6. TESTS FOR NONLINEARITY IN SERIES
P-values of Test Statistics

Variable	Bispectral	BDS Test d=2,3	Bicovariance	Tsay TAR
Industrial Production	0.359 (normal)	0.001	0.002	$<10^{-3}$
	0.288 (linear)	$<10^{-3}$		
Real Disposable Income	0.722	$<10^{-3}$	0.09	0.021
	0.094	0.00		
Real Retail Sales	0.122	$<10^{-3}$	0.004	$<10^{-3}$
	0.597	$<10^{-3}$		
Total Employment	0.019	0.003	$<10^{-3}$	0.009
	0.026	0.004		
Manufacturing Employment	0.767	$<10^{-3}$	0.01	0.001
	0.437	$<10^{-3}$		
Unemployment Rate	0.050	0.05	$<10^{-3}$	0.02
	0.780	$<10^{-3}$		
Inventory-sales Ratio	0.579	0.003	0.084	0.19
	0.291	$<10^{-3}$		
Total Capacity Utilization	0.033	$<10^{-3}$	0.202	0.007
	0.042	$<10^{-3}$		
Orders for Durable Goods	0.033	0.018	0.053	0.087
	0.616	0.001		
Orders for Industrial Machinery	0.485	$<10^{-3}$	0.049	0.311
	0.128	$<10^{-3}$		
Federal Funds Rate	0.920	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$
	0.124	$<10^{-3}$		
Commercial Paper Rate	0.806	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$
	0.087	$<10^{-3}$		
90-Day Treasury-Bill Rate	0.778	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$
	0.089	$<10^{-3}$		
Ten-Year Treasury Bond Rate	0.232	$<10^{-3}$	$<10^{-3}$	$<10^{-3}$
	0.055	$<10^{-3}$		
Real Treasury-Bill Rate	0.081	$<10^{-3}$	$<10^{-3}$	0.022
	0.033	$<10^{-3}$		
Real S&P 500 Index	0.079	0.047	0.006	0.001
	0.426	0.031		
Real Money Stock — M2	0.164	0.001	0.014	0.019
	0.919	0.002		
Money Stock — M2	0.609	$<10^{-3}$	0.085	0.11
	0.548	$<10^{-3}$		
Consumer Price Index	0.283	$<10^{-3}$	0.007	0.012
	0.901	$<10^{-3}$		

Table 7. TESTS FOR NONLINEARITY IN GARCH-ADJUSTED SERIES
P-values of Test Statistics

VARIABLE	BISPECTRAL	BDS TEST d=2,3	BICOVARIANCE	TSAY TAR
Industrial Production	0.334 (normal) 0.498 (linear)	0.671 0.624	0.572	0.407
Real Disposable Income	0.719 0.102	$<10^{-3}$ $<10^{-3}$	0.085	0.021
Real Retail Sales	0.177 0.592	0.485 0.495	0.654	0.010
Total Employment	0.152 0.089	0.249 0.373	0.166	0.567
Manufacturing Employment	0.754 0.212	0.697 0.518	0.162	0.103
Unemployment Rate	0.145 0.913	0.731 0.665	0.003	0.011
Inventory-sales Ratio	0.579 0.291	0.003 0.008	0.084	0.190
Total Capacity Utilization	0.033 0.042	$<10^{-3}$ $<10^{-3}$	0.203	0.007
Orders for Durable Goods	0.137 0.785	0.215 0.298	0.353	0.138
Orders for Industrial Machinery	0.502 0.136	$<10^{-3}$ $<10^{-3}$	0.049	0.316
Federal Funds Rate	0.145 0.113	0.475 0.66	0.24	0.087
Commercial Paper Rate	0.111 0.048	0.794 0.853	0.213	0.015
90-Day Treasury-Bill Rate	0.742 0.509	0.677 0.811	0.534	0.706
Ten-Year Treasury Bond Rate	0.292 0.071	0.858 0.878	0.324	0.859
Real Treasury-Bill Rate	0.400 0.312	0.934 0.876	0.408	0.272
Real S&P 500 Index	0.052 0.375	0.315 0.225	0.047	0.008
Real Money Stock — M2	0.170 0.903	0.008 0.004	0.015	0.023
Money Stock — M2[a]	NA	NA	NA	NA
Consumer Price Index	0.274 0.896	$<10^{-3}$ $<10^{-3}$	0.007	0.012

Table 8. Tests for the Adequacy of GARCH as a Nonlinear Representation

Variable	Tests for Independence of High Standard Deviations and Recessions						Test for Different Autoregressions
	One Month		Three Month		Six Month		
	Expected Actual	Chi-Squared p-value Likelihood Ratio p-value	Expected Actual	Chi-Squared P-Value Likelihood Ratio P-Value	Expected Actual	Chi-Squared P-Value Likelihood Ratio P-Value	p-value One-Month High Six-Month High
Industrial Production	33.0	0.034	22.9	0.077	18.5	0.111	0.632
	54.8	$<10^{-3}$	42.5	$<10^{-3}$	38.4	$<10^{-3}$	0.013
Real Disposable Income	2.8	0.502	0.7	0.748	NA[a]	NA[a]	$<10^{-3}$
	0.0	0.032	0.0	0.307			0.009
Real Retail Sales	31.8	0.037	18.2	0.115	6.2	0.358	0.534
	45.2	0.010	34.3	$<10^{-3}$	16.4	$<10^{-3}$	$<10^{-3}$
Total Employment	37.1	0.024	30.9	0.040	16.9	0.129	0.032
	43.8	0.202	38.4	0.144	15.1	0.657	$<10^{-3}$
Manufacturing Employment	27.0	0.055	16.9	0.129	11.6	0.208	$<10^{-3}$
	45.2	0.001	24.7	0.066	19.2	0.041	$<10^{-3}$
Unemployment Rate	36.9	0.025	32.0	0.036	25.7	0.061	0.001
	58.9	0.001	53.4	0.001	45.2	0.001	$<10^{-3}$
Inventory-sales Ratio	90.8	$<10^{-3}$	88.8	$<10^{-3}$	85.4	0.001	0.783
	98.6	0.003	98.6	0.001	94.5	0.009	0.964
Total Capacity Utilization	26.8	0.032	6.4	0.297	1.9	0.565	0.097
	43.6	0.002	17.7	0.001	11.3	0.001	0.005
Orders for Durable Goods	38.6	0.014	30.5	0.029	24.7	0.049	0.138
	54.8	0.002	48.0	0.001	37.0	0.010	0.164
Orders for Industrial Machinery	26.2	0.043	7.5	0.278	1.3	0.653	0.278
	43.8	0.001	11.0	0.246	4.1	0.048	0.107
Federal Funds Rate	25.1	0.056	21.8	0.074	18.2	0.102	0.022
	61.6	0.001	61.6	0.001	54.8	0.001	0.004
Commercial Paper Rate	26.2	0.054	21.2	0.085	20.4	0.091	0.027
	69.9	0.001	68.5	0.001	68.5	0.001	0.002
90-Day Treasury-Bill Rate	27.5	0.052	23.4	0.074	21.0	0.090	0.082
	71.2	0.001	68.5	0.001	64.4	0.001	0.017
Ten-Year Treasury Bond Rate	35.5	0.025	31.8	0.034	28.4	0.045	0.158
	50.7	0.004	49.3	0.001	49.3	0.001	0.079
Real Treasury-Bill Rate	33.7	0.032	23.0	0.076	19.9	0.100	0.171
	68.5	0.001	58.9	0.001	58.9	0.001	$<10^{-3}$
Real S&P 500 Index	29.6	0.044	8.4	0.283	6.4	0.351	0.148
	56.2	0.001	23.3	0.001	23.3	0.001	$<10^{-3}$
Real Money Stock — M2	26.7	0.038	6.6	0.302	NA[a]	NA[a]	$<10^{-3}$
	39.7	0.008	1.4	0.021			$<10^{-3}$
Money Stock — M2[b]	NA	NA	NA	NA	NA	NA	
Consumer Price Index	59.7	0.004	44.8	0.013	24.7	0.066	$<10^{-3}$
	65.8	0.255	56.2	0.035	34.3	0.049	0.019

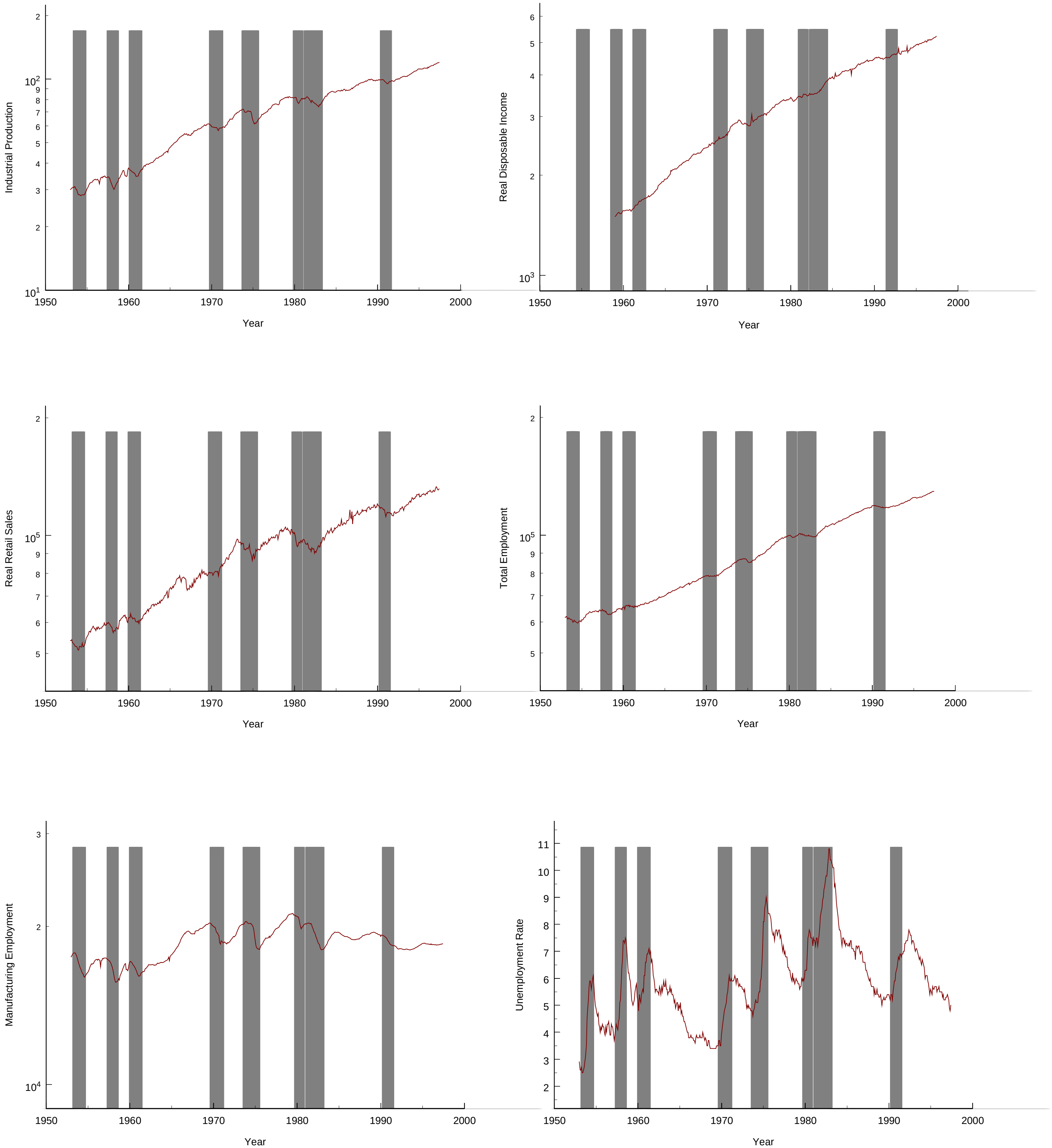
Table 9

STATISTICAL SIGNIFICANCE OF THRESHOLD AUTOREGRESSIONS

DELAY OF ONE MONTH AND THRESHOLD CHANGES OF UNEMPLOYMENT OF 0.3 AND -0.3

VARIABLE	UPPER REGION ONLY *F-RATIO(NDF1,NDF2)* *P-VALUE*	UPPER AND LOWER REGIONS *F-RATIO(NDF1,NDF2)* *P-VALUE*	LOWER REGION GIVEN UPPER REGION *F-RATIO(NDF1,NDF2)* *P-VALUE*
Industrial Production	1.653 (29,476) 0.0189	1.262 (58,447) 0.104	0.882 (29,447) 0.645
Disposable Income	2.342 (5,449) 0.0407	2.362 (10,444) 0.010	2.348 (5,444) 0.040
Real Retail Sales	1.966 (6,522) 0.0688	1.628 (12,516) 0.080	1.283 (6,516) 0.26305
Total Employment	1.652 (10,514) 0.089	1.392 (20,504) 0.120	1.127 (10,504) 0.339
Manufacturing Employment	6.524 (5,524) $<10^{-3}$	3.670 (10,519) $<10^{-3}$	0.826 (5,519) 0.532
Unemployment Rate	1.932 (27,480) 0.004	1.308 (54,453) 0.078	0.716 (27,453) 0.853
Inventory Sales Ratio	0.871 (27,480) 0.655	0.968 (54,453) 0.541	1.063 (27,453) 0.382
Total Capacity Utilization	3.660 (6,350) 0.002	2.578 (12,344) 0.003	1.466 (6,344) 0.189
Orders for Durable Goods	2.025 (9,448) 0.035	1.241 (18,439) 0.224	0.479 (9, 439) 0.889
Orders for Industrial Machinery	1.215 (9,448) 0.283	0.664 (18,439) 0.847	0.608 (9,439) 0.791
Federal Funds Rate	7.464 (19,461) $<10^{-3}$	5.191 (38,442) $<10^{-3}$	2.468 (19,442) 0.001
Commercial Paper Rate	13.920 (13,497) $<10^{-3}$	8.218 26 484 $<10^{-3}$	2.111 (13,484) 0.012
90-Day Treasury-Bill Rate	9.257 (24,486) $<10^{-3}$	7.499 (48,462) $<10^{-3}$	4.255 (24,462) $<10^{-3}$
Ten-Year Treasury Bond Rate	5.225 (18,479) $<10^{-3}$	2.937 (36,461) $<10^{-3}$	0.706 (18,461) 0.806
Real Treasury-Bill Rate	1.379 (27,480) 0.099	1.515 (54,453) 0.014	1.604 (27,453) 0.029
Real S&P 500 Index	1.471 (4,524) 0.210	1.431 (8,520) 0.181	1.386 (4,520) 0.237
Real Money Stock - M2	1.960 (11,431) 0.031	1.255 (22,420) 0.198	0.572 (11,420) 0.148
Money Stock - M2	0.896 (13,425) 0.558	1.241 (26,412) 0.194	1.571 (13,412) 0.910
Consumer Price Index	1.600 (35,464) 0.018	1.224 (70,429) 0.120	0.864 (35,429) 0.306

Figure 1
Time Series



Note: Bars denote periods defined as recessions by the NBER.

Figure 1
Time Series
(continued)

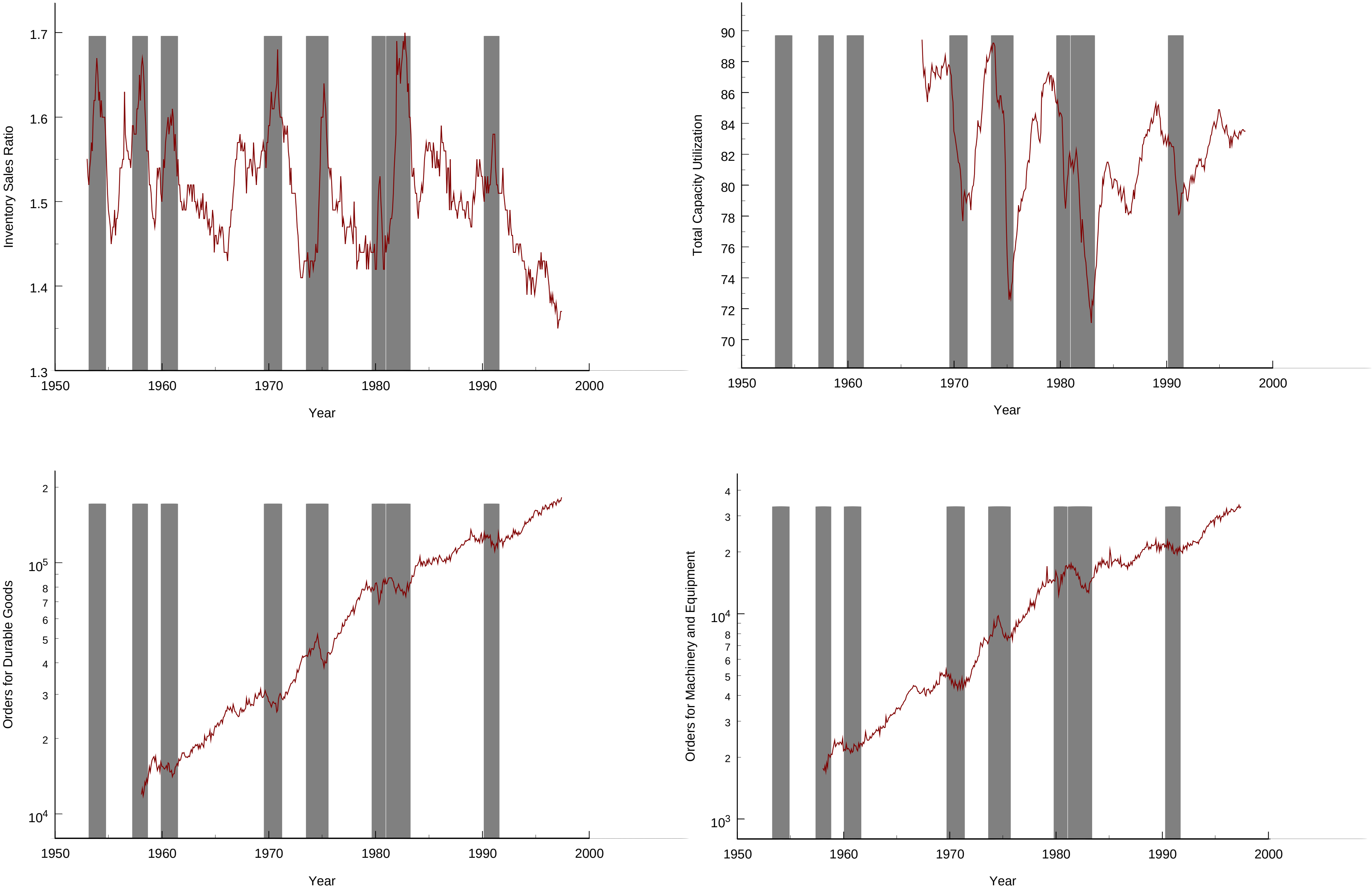
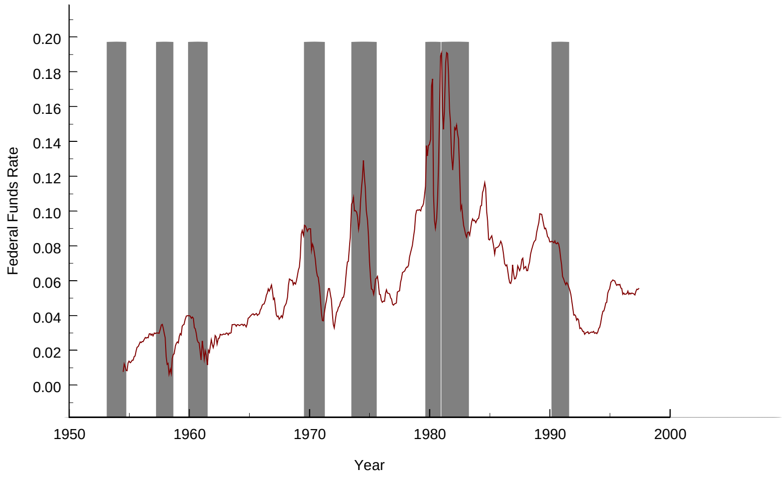
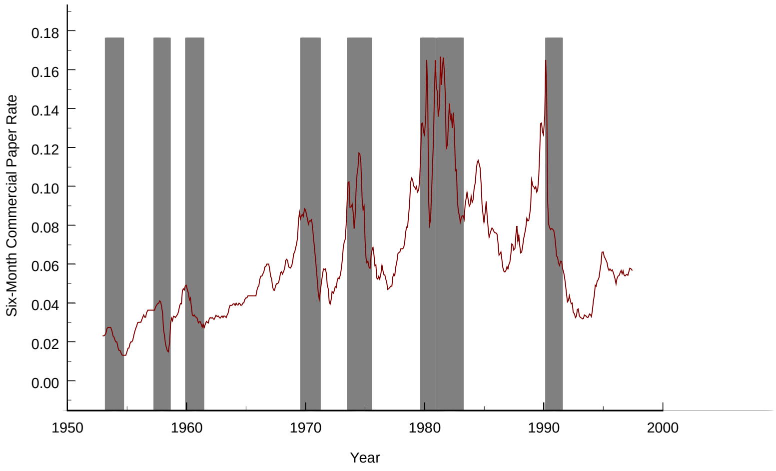
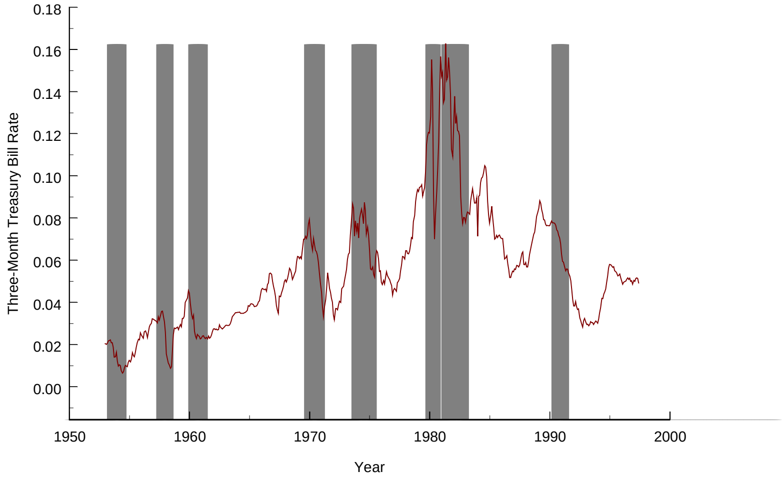
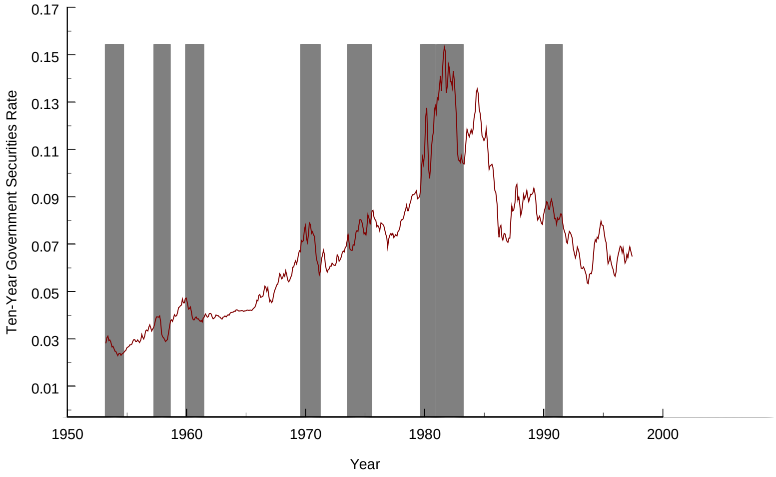


Figure 1
Time Series
(continued)









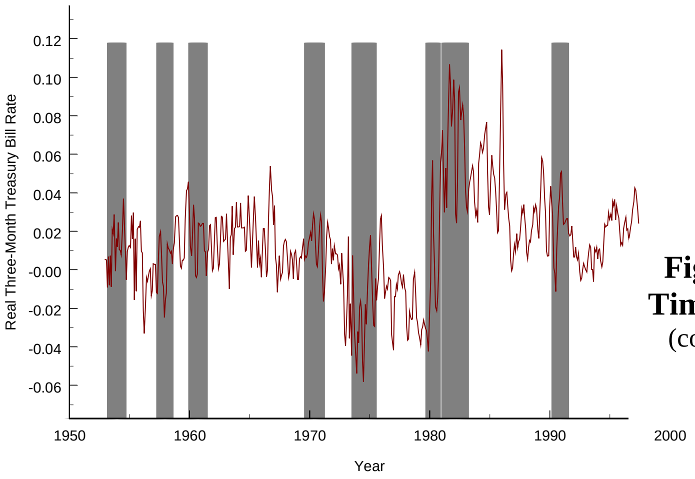


Figure 1
Time Series
(continued)

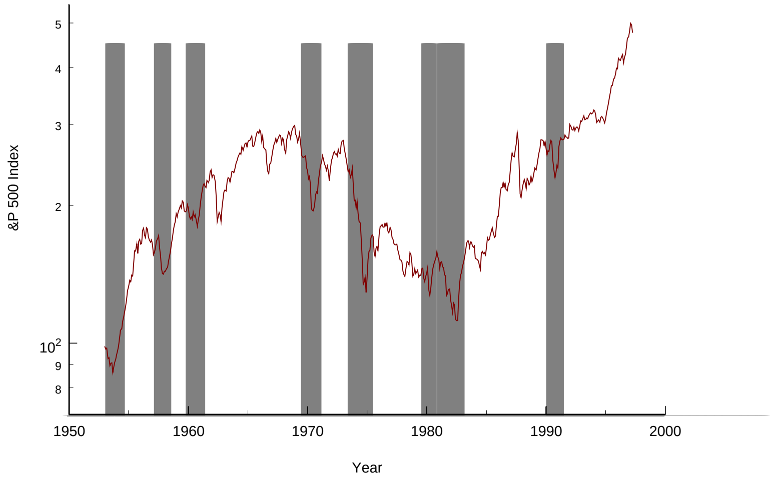


Figure 1
Time Series
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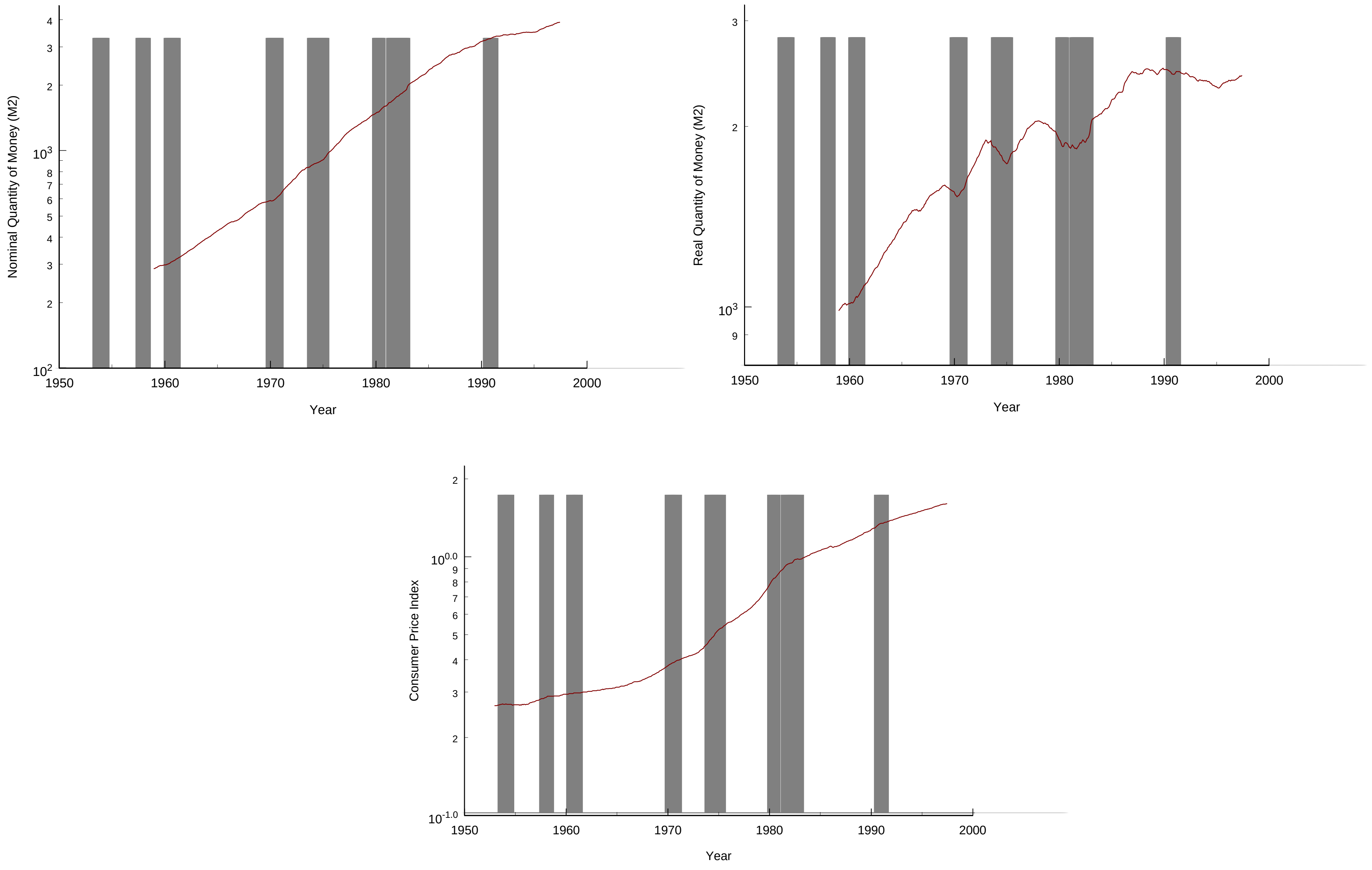

Figure 2

Residuals from the Unemployment Autoregression with GARCH

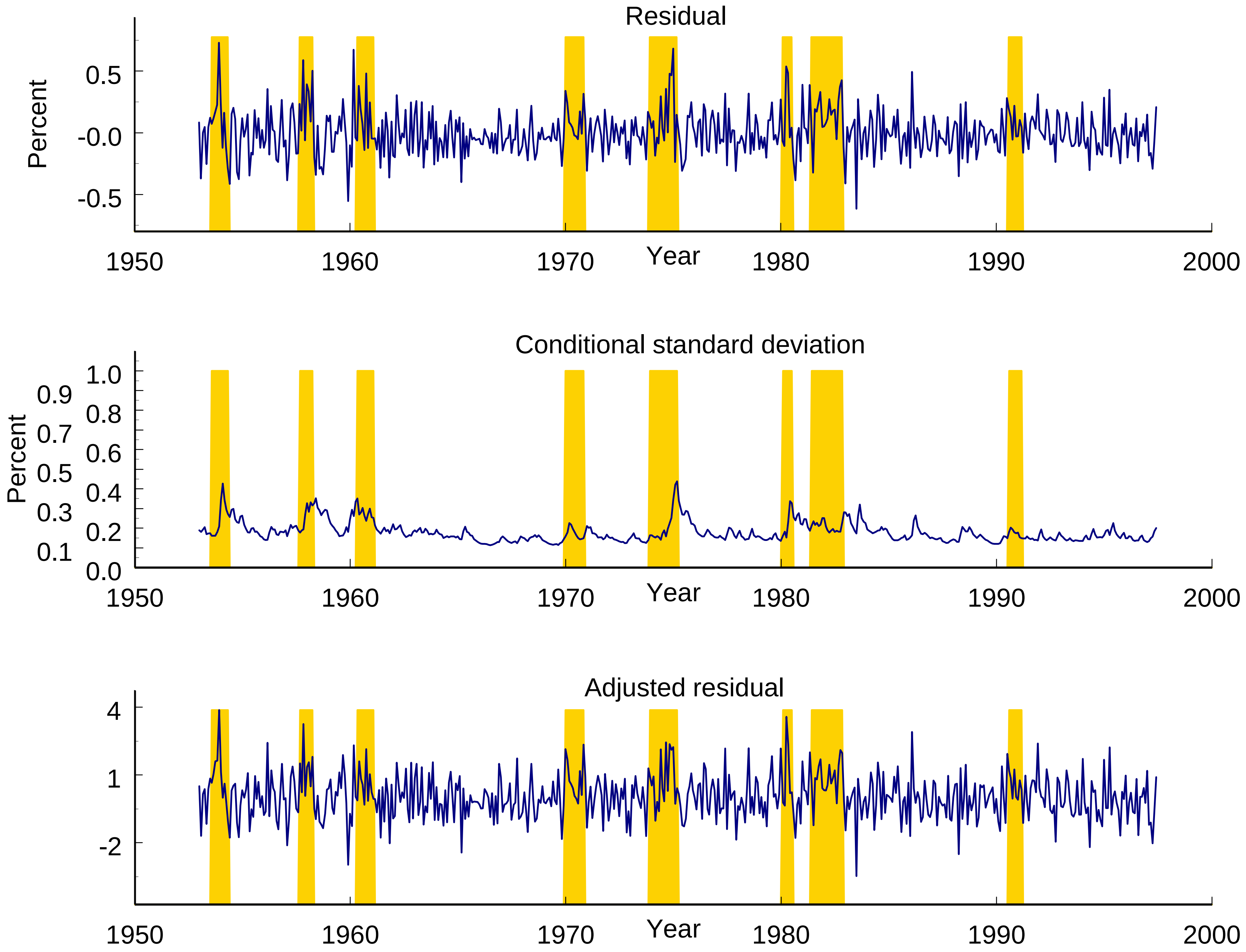

Figure 3

Two Indicators of the Aggregate Economy