

Financial Econometrics

Introduction to Financial Econometrics

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Notation for proportional returns

- p_t is the price
 - Interpret p_t as end-of-period price
 - “Price” includes all payments received
- R_t is the proportional return, $R_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1$
 - Sometimes called arithmetic return
- Gross proportional return is $\frac{p_t}{p_{t-1}} = 1 + R_t$
- $R_t[k]$ is the k -period return, $R_t[k] = \frac{p_t - p_{t-k}}{p_{t-k}} = \frac{p_t}{p_{t-k}} - 1$
- $1 + R_t[k] = \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{p_{t-2}} \dots \frac{p_{t-(k-1)}}{p_{t-k}} = \prod_{j=0}^{k-1} \frac{p_{t-j}}{p_{t-j-1}}$
 - $1 + R_t[k] = \prod_{j=0}^{k-1} (1 + R_{t-j})$

Annualized returns

- Usually annualize returns
 - If $1 + R_t [k]$ is a k -year gross return, the annualized gross return is $\left(\frac{p_t}{p_{t-k}}\right)^{1/k}$
 - The annualized net return is $\left(\frac{p_t}{p_{t-k}}\right)^{1/k} - 1$
- Often don't convert monthly or daily returns to annualized returns
 - Magnitudes would be ridiculous

Notation for logarithmic returns

- r_t is the log return, $r_t = \ln(p_t/p_{t-1}) = \ln(1 + R_t)$
 - Similar in magnitude if R_t close to zero
 - $R_t = 0.05$, $r_t = 0.0488$
 - Also can say similar in magnitude for “small” changes in price
- $r_t[k]$ is the k -period return, $r_t[k] = \ln(p_t/p_{t-k})$
- $r_t[k] = \ln(p_t/p_{t-1}) + \dots + \ln(p_{t-(k-1)}/p_{t-k}) = \sum_{j=0}^{k-1} r_{t-j}$
 - Usually annualize returns
 - If $r_t[k]$ is a k -year return, then annualized return is $r_t[k]/k$
- Can be viewed as a Taylor series approximation around zero

Log returns often handy

- Multiplication becomes addition

- $r_t [k] = \ln (p_t / p_{t-1}) + \dots + \ln (p_{t-(k-1)} / p_{t-k}) = \sum_{j=0}^{k-1} r_{t-j}$

- Lessens influence of extreme arithmetic returns

- Suppose have a set of daily data with typical arithmetic return of ± 1 percent
 - Arithmetic and log returns are about ± 1 percent
 - Suppose a couple of observation have high positive returns
 - Arithmetic return of 20 percent is about 18 percent
 - Lessens effect of observations with high returns
 - Effect bigger as arithmetic return deviates from zero
 - $p_t = 2, p_t = 1, R_t = 1$ or 100 percent
 - $r_t = \ln (2/1) = 0.693$ or 69 percent

Excess return

- Analysis often focuses on excess return
 - Not return relative to zero
- Definition: $Z_t = R_t - R_{0t}$
 - where R_{0t} is the “risk-free” arithmetic rate
 - Often written with superscript f for “risk free”
- Definition: $z_t = r_t - r_{0t}$
 - where r_{0t} is the “risk-free” log rate
 - $r_{0t} = \ln(1 + R_{0t})$

Distribution of data, e.g. returns

- Distributions
 - Joint, marginal and conditional
 - Moments of distribution, raw and about mean
- Moments of distribution about mean (except mean itself) for a series x
 - Mean

$$\hat{\mu} = \frac{\sum_{t=1}^T x_t}{T}$$

- Variance

$$\hat{\mu}_2 = \frac{\sum_{t=1}^T (x_t - \hat{\mu})^2}{T}$$

- or divided by $T - 1$
- $\hat{\mu}_2 = s$
- Standard deviation is $s = \sqrt{s^2}$

Third and fourth moments about mean

- Third moment is skewness

$$\hat{\mu}_3 = \frac{\sum_{t=1}^T (x_t - \hat{\mu})^3}{T}$$

- Roughly symmetric if skewness coefficient $\hat{\mu}_3 = 0$
 - No unequivocal measure of skewness
 - $\hat{\mu}_3 = \hat{S}(x)$
- Fourth moment is kurtosis

$$\hat{\mu}_4 = \frac{\sum_{t=1}^T (x_t - \hat{\mu})^4}{T}$$

- What is big or small?
 - Usual to measure excess kurtosis, $\hat{\mu}_4 - 3$
 - Normal distribution has $\mu_4 = 3$
 - $\hat{\mu}_4 = \hat{K}(x)$

Before doing any complex analysis of data, examine them carefully

- Illustrate with data on over 600,000 forecasts by analysts of firms's earnings
 - Interesting partly because maybe forecast “surprises” may affect stock price
 - Earnings greater than expected increase stock price if result in forecast of higher earnings in the future
 - Earnings greater than expected decrease stock price if result in forecast of higher earnings in the future
- Analyze earnings surprise

$$e_{T,t}^{i,j} = \frac{a_T^i - f_{T,t}^{i,j}}{p_{T-1}^i}$$

where a_T^i is the earnings announcement for firm i at time T , $f_{T,t}^{i,j}$ is the forecast made for firm i 's earnings at T by analyst j , with forecast made at time t (before T) and p_{T-1}^i is the stock price for firm i at $T - 1$ (before T)

Characteristics of earnings surprise data

- Data from “Investment Analysts’ Forecasts of Earnings” by Rocco Ciceretti, Iftekhar Hasan and me
- Clean up data
 - Look for apparent errors (e.g. earnings many times greater than stock price)
 - Restrict to forecasts of U.S. firms by U.S. analysts
 - End up with 662,016 observations for 6,574 companies
- Might think we can’t “look” at these data

Statistical summary of data

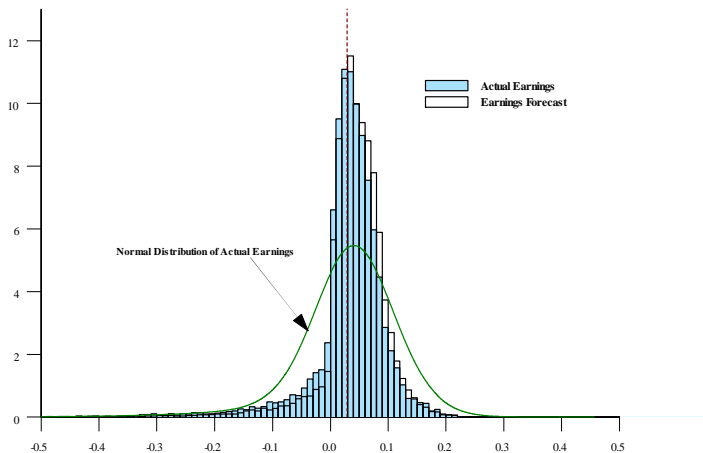
Summary Table 1

- Survey Table 1.pdf

Graphical summary of data for twelve-month-ahead forecasts

Figure 1
Actual Earnings and Earnings Forecast

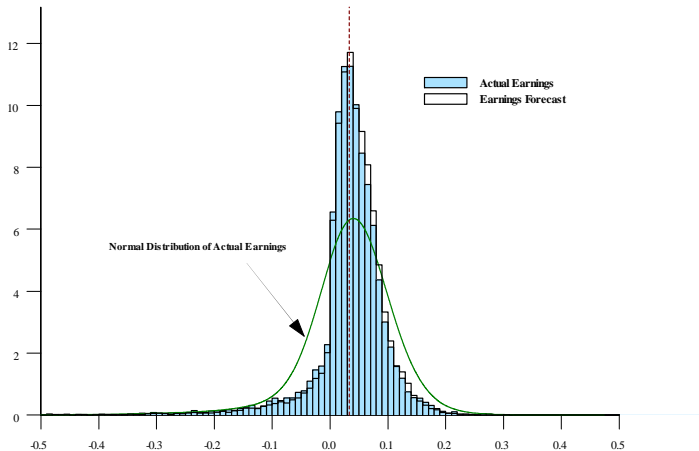
Panel 3: Forecast Horizon of 12 Months



Graphical summary of data for six-month-ahead forecasts

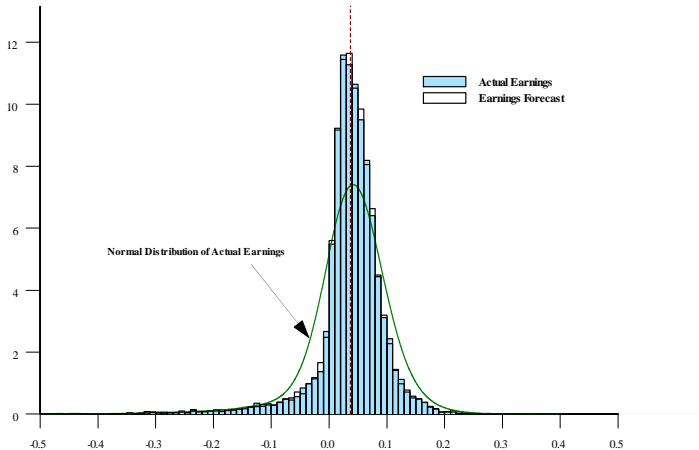
Figure 1
Actual Earnings and Earnings Forecast

Panel 2: Forecast Horizon of 6 Months



Graphical summary of data for one-month-ahead forecasts

Figure 1
Actual Earnings and Earnings Forecast
Panel I: Forecast Horizon of One Month



Summary statistics for twelve-month-ahead forecasts

Summary Table 2 page 3

Summary statistics for six-month-ahead forecasts

Summary Table 2 page 2

Summary statistics for one-month-ahead forecasts

Summary Table 2 page 1

Returns distribution – Independent and identical normal distribution is simple

- Likelihood function

$$L(r_t|\theta) = \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{(r_t - \mu)^2}{2\sigma_t^2}\right)$$

- Note time-varying variance
- With constant variance
 - Advantages: Simple and computationally tractable
 - Disadvantages : Not really consistent with the data
- Distributions more consistent with the data?
 - Time-varying variance
 - Depending on time frame, returns are not independent over time
 - r_t is correlated with r_{t-1}
 - Correlation changes with time frame (minutes, versus days, versus months or years)

Empirical analysis of returns on stock indices and two individual stocks

- CRSP value-weighted and equally-weighted indices
- IBM
- Amazon
- Returns and volatility of returns