

# Financial Econometrics

## Introduction to Financial Econometrics

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# Notation for proportional returns

- $p_t$  is the price
  - Interpret  $p_t$  as end-of-period price
  - “Price” includes all payments received
- $R_t$  is the proportional return,  $R_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1$ 
  - Sometimes called arithmetic return
- Gross proportional return is  $\frac{p_t}{p_{t-1}} = 1 + R_t$
- $R_t[k]$  is the  $k$ -period return,  $R_t[k] = \frac{p_t - p_{t-k}}{p_{t-k}} = \frac{p_t}{p_{t-k}} - 1$
- $1 + R_t[k] = \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{p_{t-2}} \dots \frac{p_{t-(k-1)}}{p_{t-k}} = \prod_{j=0}^{k-1} \frac{p_{t-j}}{p_{t-j-1}}$ 
  - $1 + R_t[k] = \prod_{j=0}^{k-1} (1 + R_{t-j})$

# Annualized returns

- Usually annualize returns
  - If  $1 + R_t [k]$  is a  $k$ -year gross return, the annualized gross return is  $\left(\frac{p_t}{p_{t-k}}\right)^{1/k}$
  - The annualized net return is  $\left(\frac{p_t}{p_{t-k}}\right)^{1/k} - 1$
- Often don't convert monthly or daily returns to annualized returns
  - Magnitudes would be ridiculous

# Notation for logarithmic returns

- $r_t$  is the log return,  $r_t = \ln(p_t/p_{t-1}) = \ln(1 + R_t)$ 
  - Similar in magnitude if  $R_t$  close to zero
  - $R_t = 0.05$ ,  $r_t = 0.0488$
  - Also can say similar in magnitude for “small” changes in price
- $r_t[k]$  is the  $k$ -period return,  $r_t[k] = \ln(p_t/p_{t-k})$
- $r_t[k] = \ln(p_t/p_{t-1}) + \dots + \ln(p_{t-(k-1)}/p_{t-k}) = \sum_{j=0}^{k-1} r_{t-j}$ 
  - Usually annualize returns
  - If  $r_t[k]$  is a  $k$ -year return, then annualized return is  $r_t[k] / k$
- Can be viewed as a Taylor series approximation around zero

# Log returns often handy

- Multiplication becomes addition

- $r_t [k] = \ln(p_t/p_{t-1}) + \dots + \ln(p_{t-(k-1)}/p_{t-k}) = \sum_{j=0}^{k-1} r_{t-j}$

- Lessens influence of extreme arithmetic returns

- Suppose have a set of daily data with typical arithmetic return of  $\pm 1$  percent
    - Arithmetic and log returns are about  $\pm 1$  percent
  - Suppose a couple of observation have high positive returns
    - Arithmetic return of 20 percent is log return of about 18 percent
    - Lessens effect of observations with high returns
  - Effect bigger as arithmetic return deviates from zero
    - $p_t = 2, p_t = 1, R_t = 1$  or 100 percent
    - $r_t = \ln(2/1) = 0.693$  or 69 percent

# Excess return

- Analysis often focuses on excess return
  - Not return relative to zero
- Definition:  $Z_t = R_t - R_{0t}$ 
  - where  $R_{0t}$  is the “risk-free” arithmetic rate
  - Often written with superscript  $f$  for “risk free”
- Definition:  $z_t = r_t - r_{0t}$ 
  - where  $r_{0t}$  is the “risk-free” log rate
  - $r_{0t} = \ln(1 + R_{0t})$

# Distribution of data, e.g. returns

- Distributions
  - Joint, marginal and conditional
  - Moments of distribution, raw and about mean
- Moments of distribution about mean (except mean itself) for a series  $x$ 
  - Mean

$$\hat{\mu} = \frac{\sum_{t=1}^T x_t}{T}$$

- Variance

$$\hat{\mu}_2 = s^2 = \frac{\sum_{t=1}^T (x_t - \hat{\mu})^2}{T}$$

- or divided by  $T - 1$
- $\hat{\mu}_2^{1/2} = s$
- Standard deviation is  $s = \sqrt{s^2}$

# Third and fourth moments about mean

- Third moment measures skewness

$$\hat{\mu}_3 = \frac{\sum_{t=1}^T (x_t - \hat{\mu})^3}{T}$$

- Roughly symmetric if skewness coefficient  $\hat{\mu}_3 = 0$
  - No unequivocal measure of skewness
  - $\hat{\mu}_3 = \hat{S}(x)$
- Fourth moment measures kurtosis

$$\hat{\mu}_4 = \frac{\sum_{t=1}^T (x_t - \hat{\mu})^4}{T}$$

- What is big or small?
  - Usual to measure excess kurtosis,  $\hat{\mu}_4 - 3$
  - Normal distribution has  $\mu_4 = 3$
  - $\hat{\mu}_4 = \hat{K}(x)$

# Before doing any complex analysis of data, examine them carefully

- Illustrate with data on over 600,000 forecasts by analysts of firms's earnings
  - Interesting partly because maybe forecast “surprises” may affect stock price
    - Earnings greater than expected increase stock price if result in forecast of higher earnings in the future
    - Earnings less than expected decrease stock price if result in forecast of lower earnings in the future
- Analyze earnings surprise

$$e_{T,t}^{i,j} = \frac{a_T^i - f_{T,t}^{i,j}}{p_{T-1}^i}$$

where  $a_T^i$  is the earnings announcement for firm  $i$  at time  $T$ ,  $f_{T,t}^{i,j}$  is the forecast made for firm  $i$ 's earnings at  $T$  by analyst  $j$ , with forecast made at time  $t$  (before  $T$ ) and  $p_{T-1}^i$  is the stock price for firm  $i$  at  $T - 1$  (before  $T$ )

# Characteristics of earnings surprise data

- Data from “Investment Analysts’ Forecasts of Earnings” by Rocco Ciceretti, Iftekhar Hasan and me
- Clean up data
  - Look for apparent errors (e.g. earnings many times greater than stock price)
  - Restrict to forecasts of U.S. firms by U.S. analysts
  - End up with 662,016 observations for 6,574 companies
- Might think we can’t “look” at these data

# Statistical summary of data

## Summary Table 1

- Survey Table 1.pdf

Table 1  
Summary of Minimum and Maximum Values and Observations Suppressed in Figures 1 and 2

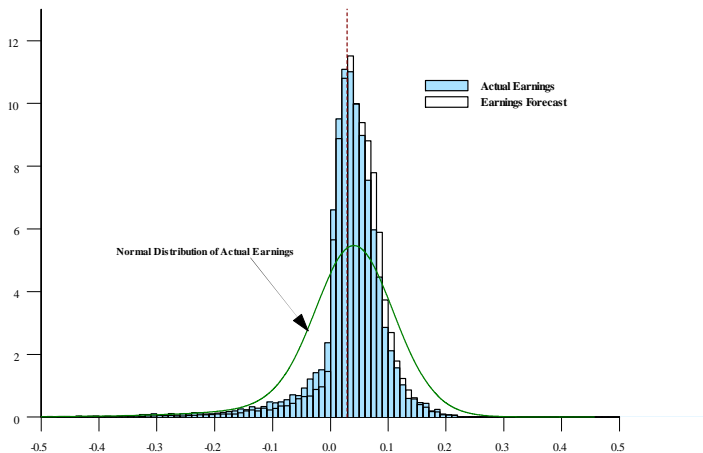
Variable	<u>Twelve Month Horizon</u>			<u>Six Month Horizon</u>			<u>One Month Horizon</u>		
	Minimum	Maximum	Number of Suppressed Observations	Minimum	Maximum	Number of Suppressed Observations	Minimum	Maximum	Number of Suppressed Observations
Actual Earnings	-1.6137	0.2844	150	-1.1820	0.3350	58	-0.9026	0.2844	11
Earnings Forecasts	-1.1532	0.2933	76	-0.7732	0.3267	21	-0.6487	0.2778	10
Forecast Errors	-1.2442	0.7614	89	-1.1561	0.5533	15	-0.6085	0.3531	2

Note: For actual earnings and earnings forecasts there are no positive observations outside the -0.5 to +0.5 range. For forecast errors, there are 6, 2 and 0 excluded positive observations at the 12, 6, and 1 forecast horizon; the remaining are negative.

# Graphical summary of data for twelve-month-ahead forecasts

**Figure 1**  
**Actual Earnings and Earnings Forecast**

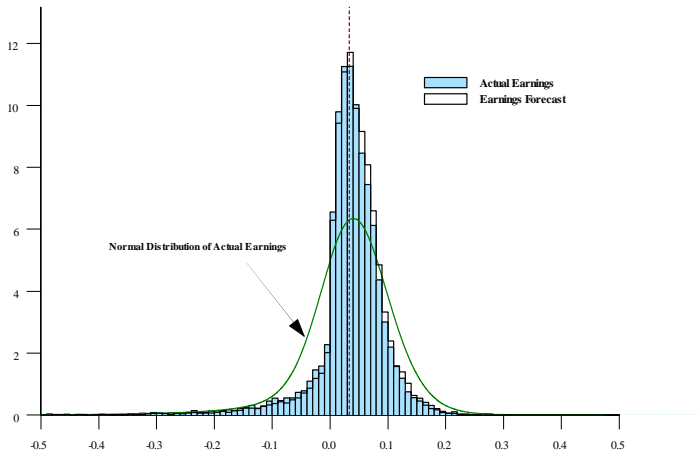
Panel 3: Forecast Horizon of 12 Months



# Graphical summary of data for six-month-ahead forecasts

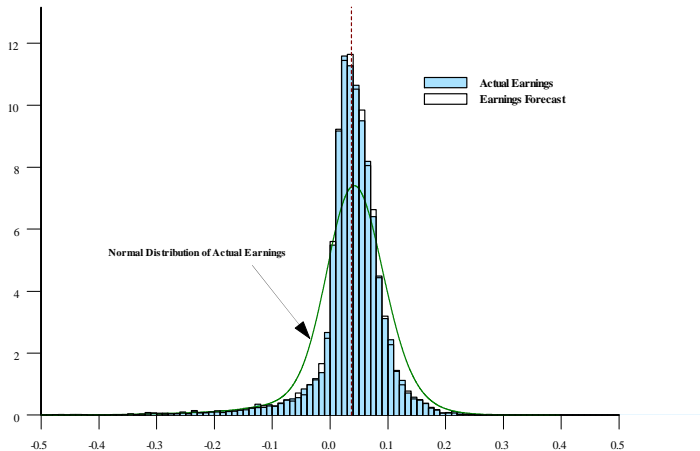
**Figure 1**  
**Actual Earnings and Earnings Forecast**

Panel 2: Forecast Horizon of 6 Months



# Graphical summary of data for one-month-ahead forecasts

**Figure 1**  
**Actual Earnings and Earnings Forecast**  
Panel I: Forecast Horizon of One Month



# Summary statistics for twelve-month-ahead forecasts

Survey Table 2

Table 2  
Distribution of Forecast Errors by Year and Horizon  
Twelve Month Horizon

	Minimum	1%	5%	10%	25%	Median	75%	90%	95%	99%	Maximum	Mean	Standard Deviation	Skewness Coefficient	Kurtosis
1990	-.81	-.4278	-.1265	-.0721	-.0249	-.0040	.0003	.0059	.0121	.0456	.09	-.0270	.0754	-4.98	31.33
1991	-.88	-.3711	-.1320	-.0770	-.0245	-.0048	.0002	.0068	.0177	.0667	.30	-.0249	.0711	-4.95	37.73
1992	-.40	-.2019	-.0922	-.0509	-.0158	-.0023	.0012	.0098	.0193	.0557	.12	-.0141	.0418	-3.53	18.96
1993	-.38	-.1789	-.0649	-.0367	-.0110	-.0011	.0022	.0088	.0185	.0636	.11	-.0095	.0368	-3.69	22.69
1994	-.47	-.1807	-.0629	-.0334	-.0091	-.0003	.0024	.0100	.0194	.0554	.17	-.0096	.0431	-6.08	52.96
1995	-.27	-.1297	-.0618	-.0367	-.0099	.0000	.0039	.0118	.0201	.0633	.18	-.0071	.0309	-2.50	16.08
1996	-.29	-.1455	-.0697	-.0379	-.0100	-.0001	.0032	.0134	.0256	.0593	.20	-.0078	.0337	-2.20	13.34
1997	-.45	-.1566	-.0608	-.0329	-.0093	-.0008	.0023	.0085	.0143	.0400	.11	-.0094	.0362	-5.56	49.00
1998	-.49	-.2378	-.0704	-.0495	-.0198	-.0035	.0010	.0060	.0131	.0419	.27	-.0154	.0422	-4.19	29.79
1999	-.76	-.2484	-.0743	-.0391	-.0119	.0000	.0050	.0224	.0430	.1306	.39	-.0079	.0576	-3.74	39.19
2000	-.51	-.2230	-.0752	-.0395	-.0120	.0003	.0055	.0276	.0634	.1277	.31	-.0054	.0508	-2.41	17.01
2001	-1.24	-.3840	-.1364	-.0785	-.0335	-.0086	.0007	.0091	.0208	.1803	.76	-.0265	.0895	-4.00	50.19
2002	-.74	-.2228	-.0656	-.0370	-.0114	-.0002	.0064	.0234	.0426	.0976	.32	-.0067	.0522	-5.09	53.33
2003	-.71	-.1839	-.0617	-.0339	-.0104	.0003	.0092	.0266	.0443	.0949	.28	-.0045	.0464	-3.98	38.24
2004	-.33	-.1148	-.0438	-.0212	-.0068	.0010	.0088	.0264	.0394	.0812	.14	-.0003	.0317	-3.10	26.77

Six Month Horizon

	Minimum	1%	5%	10%	25%	Median	75%	90%	95%	99%	Maximum	Mean	Standard Deviation	Skewness Coefficient	Kurtosis
1990	-1.16	-.2730	-.0955	-.0427	-.0122	-.0016	.0008	.0060	.0142	.0575	.20	-.0162	.0669	-7.95	92.95
1991	-.54	-.2171	-.0642	-.0353	-.0097	-.0015	.0009	.0074	.0176	.0600	.18	-.0108	.0441	-5.33	44.17
1992	-.32	-.1301	-.0444	-.0219	-.0071	-.0006	.0013	.0062	.0122	.0357	.11	-.0066	.0276	-5.01	39.50
1993	-.16	-.0814	-.0247	-.0137	-.0037	-.0001	.0018	.0066	.0142	.0409	.18	-.0024	.0181	-2.34	24.80
1994	-.17	-.0705	-.0284	-.0159	-.0041	.0000	.0024	.0076	.0129	.0400	.16	-.0025	.0170	-1.96	20.70
1995	-.30	-.0828	-.0330	-.0169	-.0044	.0000	.0022	.0065	.0111	.0293	.10	-.0038	.0198	-5.37	52.00
1996	-.32	-.0969	-.0287	-.0152	-.0038	.0001	.0024	.0090	.0151	.0389	.19	-.0029	.0227	-4.78	54.34
1997	-.27	-.0907	-.0275	-.0132	-.0030	.0001	.0023	.0079	.0146	.0422	.17	-.0021	.0206	-2.77	38.07
1998	-.33	-.0992	-.0359	-.0219	-.0081	-.0016	.0008	.0043	.0094	.0290	.29	-.0063	.0226	-3.18	49.61
1999	-.56	-.1600	-.0446	-.0202	-.0048	.0001	.0031	.0109	.0193	.0533	.55	-.0052	.0383	-3.74	78.39
2000	-.36	-.1101	-.0447	-.0221	-.0059	.0000	.0022	.0136	.0261	.0668	.17	-.0037	.0273	-2.68	26.48
2001	-.64	-.1714	-.0494	-.0274	-.0092	-.0015	.0012	.0074	.0141	.0581	.20	-.0085	.0391	-5.95	66.46
2002	-.38	-.0997	-.0325	-.0158	-.0054	-.0003	.0027	.0088	.0159	.0402	.21	-.0038	.0269	-6.09	76.24
2003	-.49	-.0994	-.0295	-.0140	-.0036	.0004	.0045	.0125	.0213	.0667	.38	-.0011	.0310	-2.52	68.31
2004	-.29	-.0617	-.0284	-.0184	-.0045	.0000	.0032	.0092	.0164	.0389	.09	-.0025	.0195	-5.05	57.05

One Month Horizon

	Minimum	1%	5%	10%	25%	Median	75%	90%	95%	99%	Maximum	Mean	Standard Deviation	Skewness Coefficient	Kurtosis
1990	-.61	-.0970	-.0286	-.0146	-.0031	-.0001	.0014	.0054	.0131	.0526	.22	-.0035	.0342	-11.48	204.59
1991	-.24	-.0659	-.0231	-.0111	-.0024	.0000	.0020	.0074	.0141	.0395	.13	-.0015	.0188	-2.99	48.29
1992	-.14	-.0698	-.0118	-.0053	-.0010	.0002	.0025	.0073	.0144	.0402	.24	.0006	.0220	4.09	61.43
1993	-.26	-.0659	-.0127	-.0064	-.0012	.0001	.0020	.0062	.0112	.0400	.10	-.0005	.0154	-4.97	71.88
1994	-.11	-.0274	-.0079	-.0039	-.0007	.0002	.0020	.0057	.0104	.0289	.09	.0006	.0102	-1.20	41.64
1995	-.22	-.0455	-.0093	-.0048	-.0009	.0002	.0019	.0057	.0114	.0390	.31	.0004	.0188	1.28	104.42
1996	-.20	-.0277	-.0078	-.0036	-.0005	.0003	.0017	.0054	.0097	.0482	.17	.0008	.0137	-.90	89.84
1997	-.36	-.0375	-.0114	-.0047	-.0006	.0003	.0019	.0054	.0096	.0325	.19	.0002	.0145	-6.48	217.27
1998	-.16	-.0256	-.0089	-.0044	-.0006	.0003	.0017	.0050	.0089	.0285	.20	.0004	.0102	1.12	110.97
1999	-.23	-.0410	-.0069	-.0031	-.0004	.0004	.0023	.0062	.0116	.0457	.28	.0011	.0158	1.31	118.62
2000	-.24	-.0673	-.0141	-.0057	-.0007	.0002	.0013	.0044	.0088	.0291	.11	-.0011	.0147	-6.61	83.84
2001	-.18	-.0371	-.0101	-.0038	-.0005	.0002	.0014	.0038	.0066	.0211	.08	-.0004	.0104	-6.24	94.52
2002	-.26	-.0340	-.0079	-.0036	-.0005	.0003	.0013	.0038	.0067	.0211	.35	-.0002	.0135	.63	268.15
2003	-.36	-.0645	-.0100	-.0047	-.0007	.0003	.0018	.0054	.0097	.0373	.15	-.0003	.0157	-7.81	145.27
2004	-.15	-.0333	-.0078	-.0037	-.0007	.0004	.0022	.0052	.0087	.0255	.15	.0006	.0092	.89	77.55

\* This test statistic has a Chi-square distribution with two degrees of freedom under the null hypothesis. The value of this Chi-square at the .001 level of significance is 13.8. All of the values in the table have  $p$ -values less than  $10^{-8}$ .

# Returns distribution – Independent and identical normal distribution is simple

- Likelihood function

$$L(r_t|\theta) = \prod_{t=1}^T \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{(r_t - \mu)^2}{2\sigma_t^2}\right)$$

- Note time-varying variance
- With constant variance
  - Advantages: Simple and computationally tractable
  - Disadvantages : Not really consistent with the data
- Distributions more consistent with the data?
  - Time-varying variance
  - Depending on time frame, returns are not independent over time
    - $r_t$  is correlated with  $r_{t-1}$
    - Correlation changes with time frame (minutes, versus days, versus months or years)

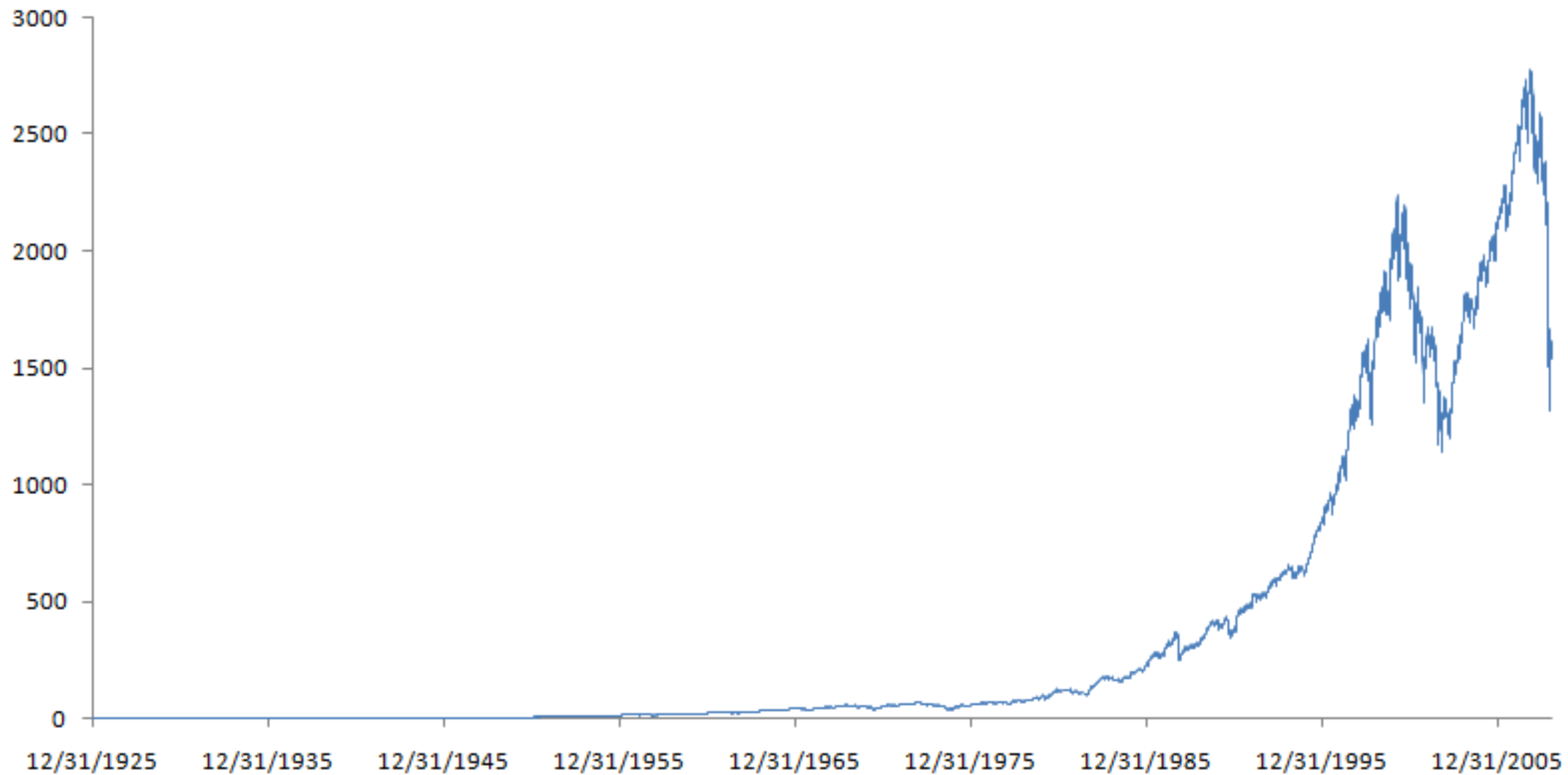
# Empirical analysis of returns on stock index

- CRSP value-weighted index
- With dividends reinvested
- Returns and volatility of returns

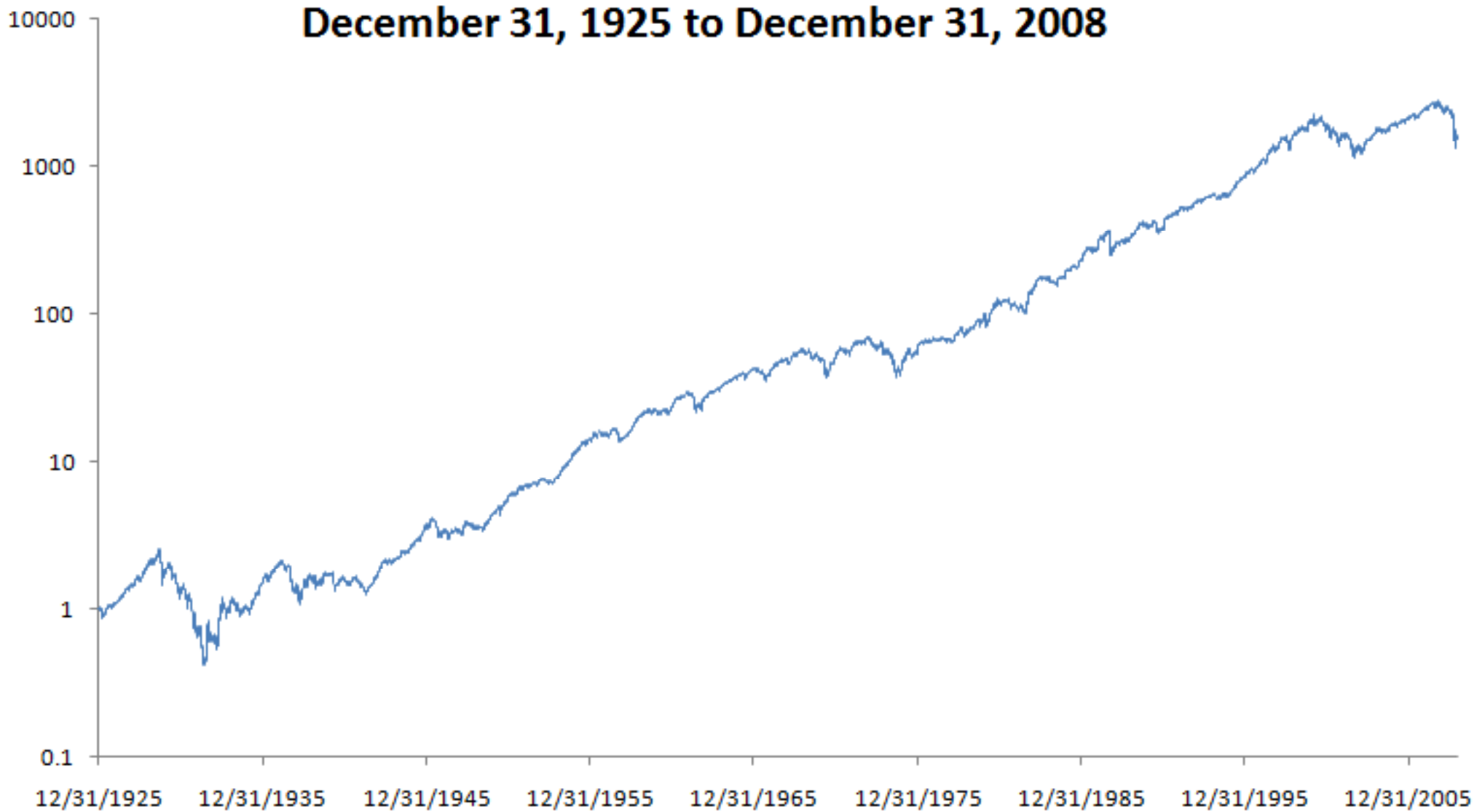
# CRSP value-weighted index with dividends reinvested

- VWCRSPDaily.pdf
- VWCRSPDailyLog.pdf
- VWCRSPMonthly.pdf
- VWRETDDaily.pdf
- VWRETDMonthly.pdf
- DCRSPMonthly.pdf
- DLCRSPMontly.pdf
- DLCRSP\_Sq.pdf
- DLCRSP\_Abs.pdf

## CRSP daily index with dividends reinvested December 31, 1925 to December 31, 2008

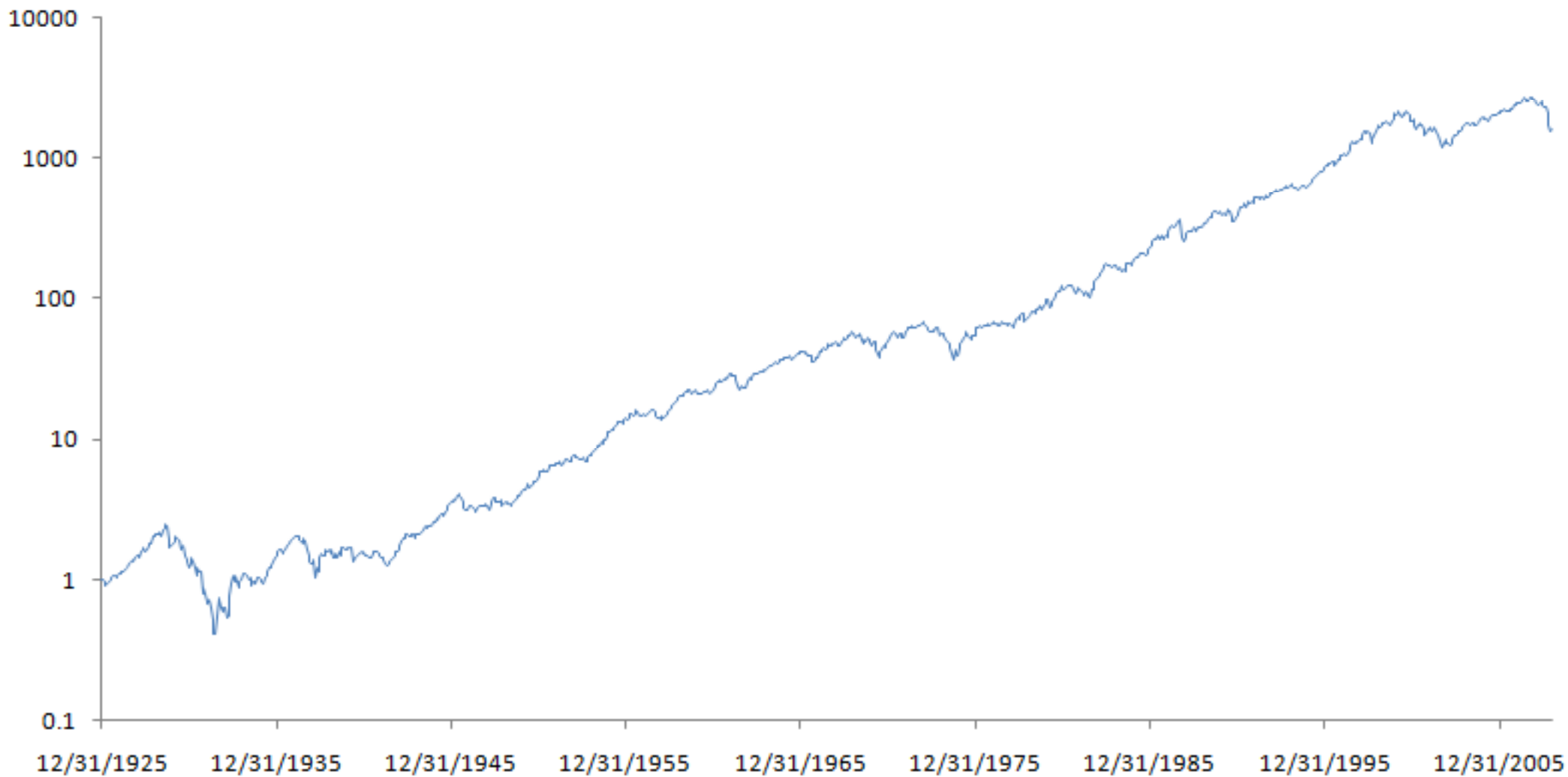


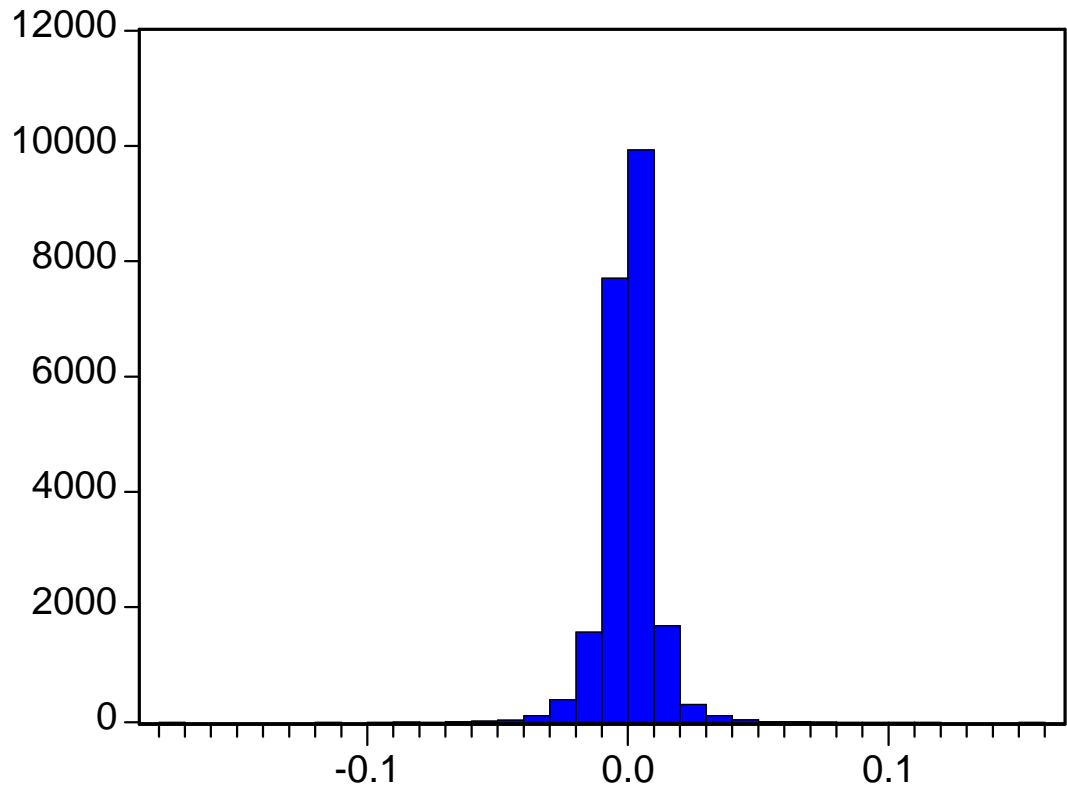
# CRSP daily index with dividends reinvested December 31, 1925 to December 31, 2008



# CRSP monthly index with dividends reinvested

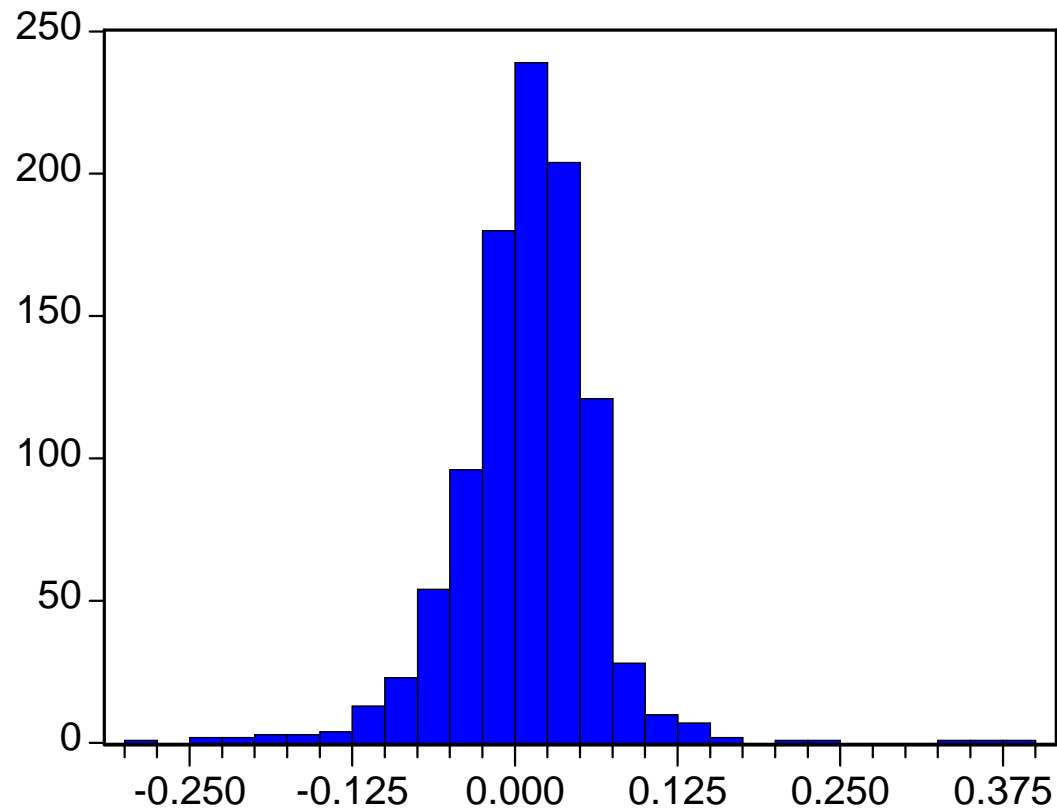
December 31, 1925 to December 31, 2008





Series: VWRETD  
Sample 1 22025  
Observations 22025

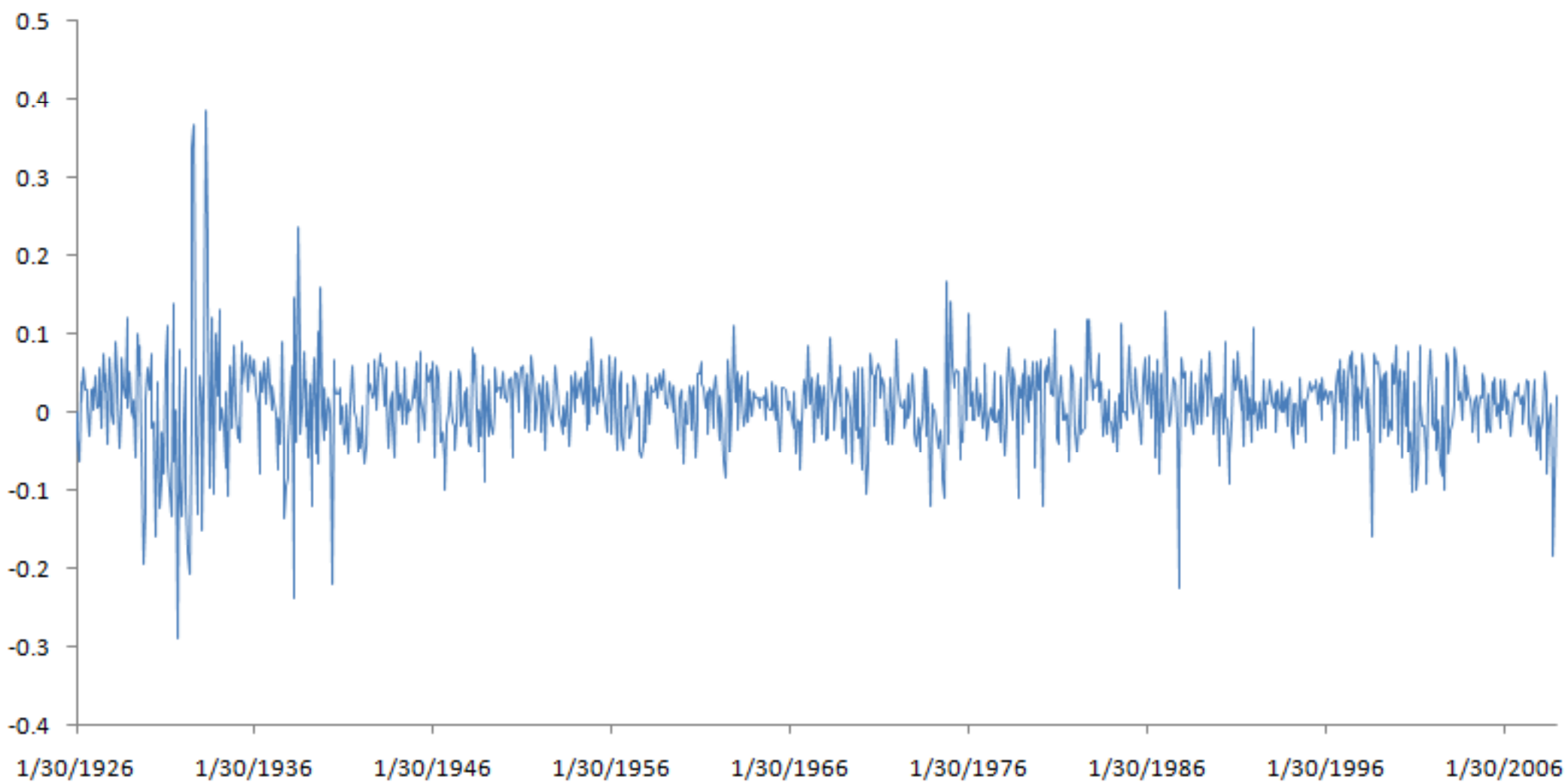
Mean	0.000391
Median	0.000751
Maximum	0.155045
Minimum	-0.171349
Std. Dev.	0.010514
Skewness	-0.139855
Kurtosis	20.93535
Jarque-Bera	295277.3
Probability	0.000000



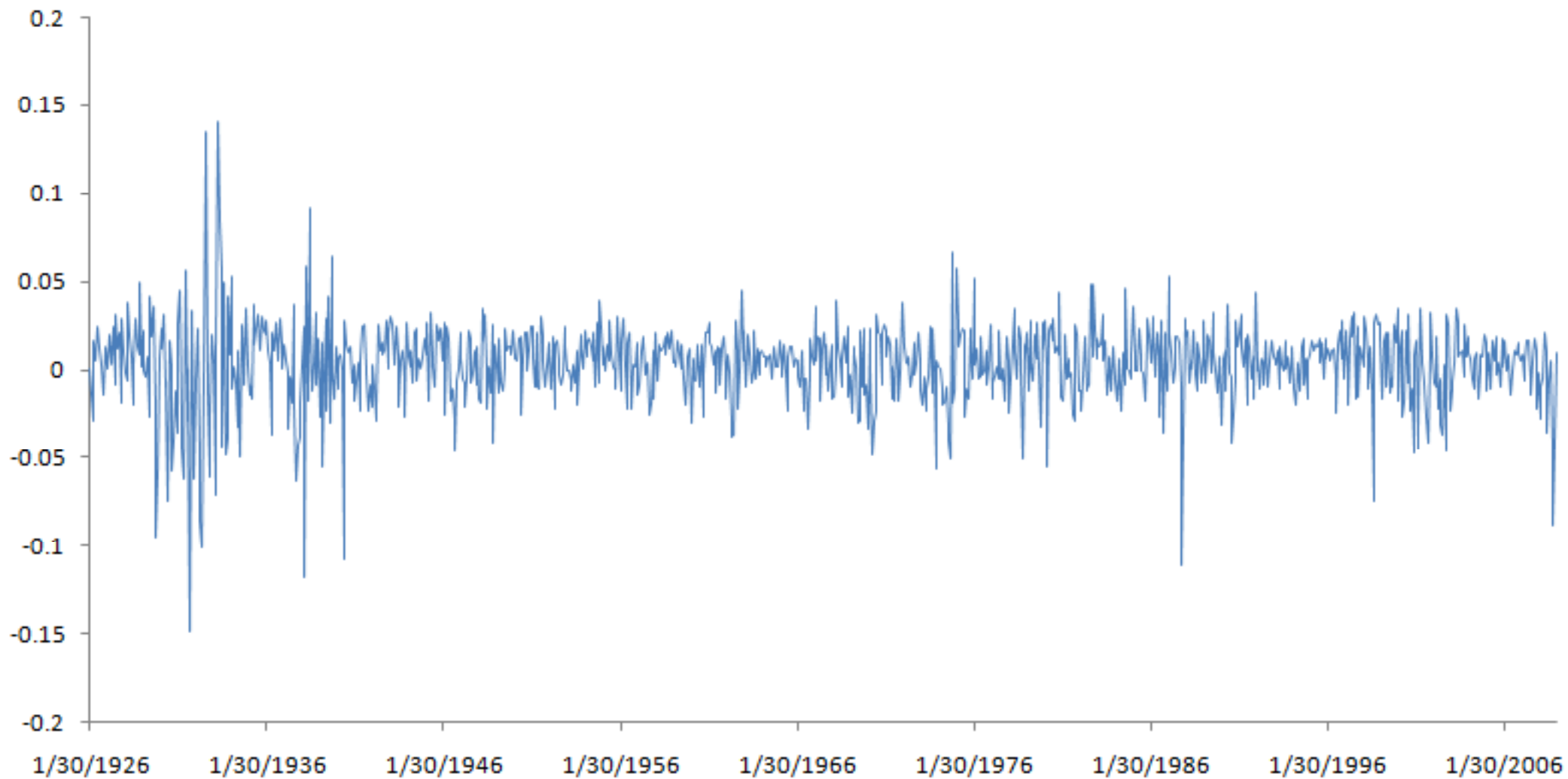
Series: VWRET  
Sample 1925:12 2008:12  
Observations 997

Mean	0.008895
Median	0.012608
Maximum	0.383683
Minimum	-0.290070
Std. Dev.	0.054298
Skewness	0.145829
Kurtosis	10.72530
Jarque-Bera	2482.748
Probability	0.000000

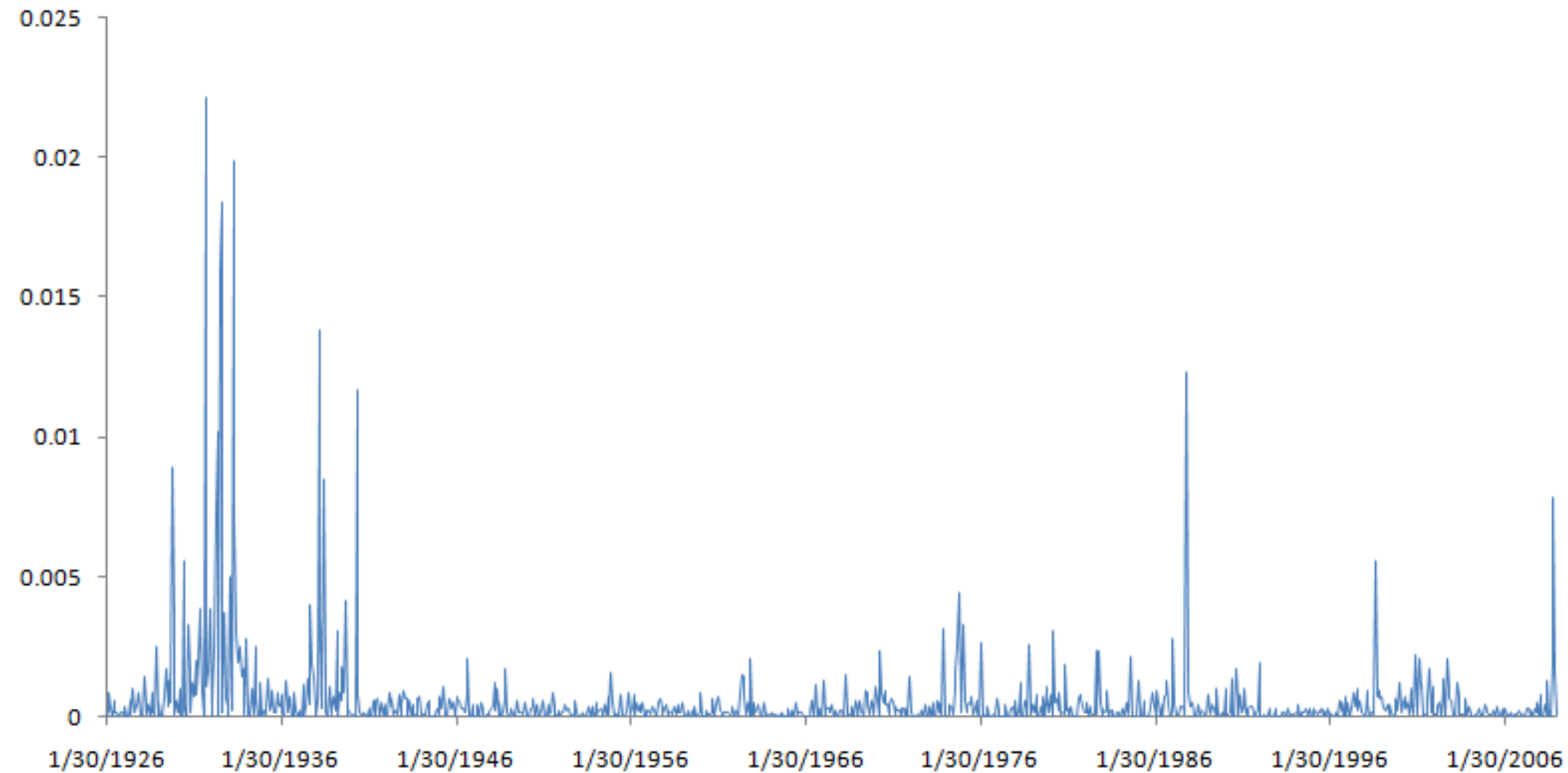
## Percent change CRSP index with dividends reinvested January 31, 1926 to December 31, 2008



## Change in logarithm of CRSP index January 31, 1926 to December 31, 2008



## Squared changes in log of CRSP index January 31, 1926 to December 31, 2008



## Absolute value of changes in log of CRSP index January 31, 1926 to December 31, 2008

