

Financial Econometrics

Long-run Relationships in Finance

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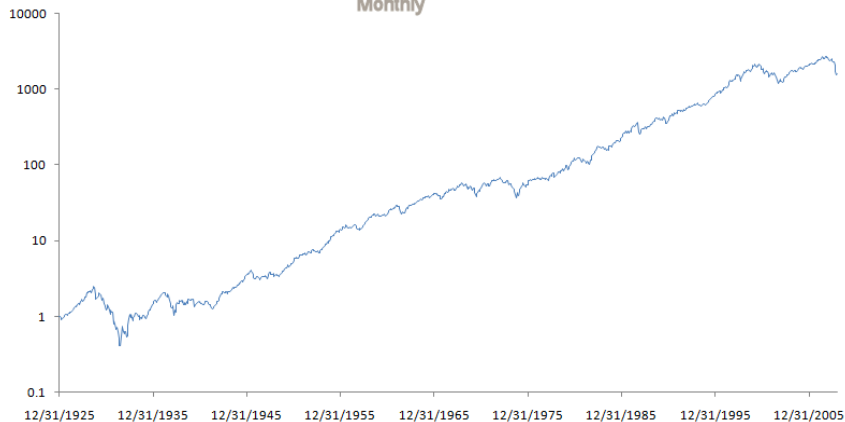
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S&P 500 Monthly



Value-weighted CRSP level with dividends reinvested monthly

CRSP monthly index with dividends reinvested
December 31, 1925 to December 31, 2008
Monthly



What can we infer from graph?

- No evidence of a constant mean of level of stock prices in 82 years
- Not likely to have a stationary representation of level
 - Not a constant mean
- Generally goes up but not always
- Unit root nonstationarity or trend?

Nonstationarity in mean

- Two types of nonstationarity in mean usually considered
 - Unit-root
 - Trend
- Unit root nonstationary (difference stationary)
 - $y_t - y_{t-1}$ is stationary
- Trend stationary
 - $y_t - \beta t$ is stationary

Unit root tests with trend

Dickey-Fuller test

- Dickey-Fuller test with drift or trend
 - Trend in level versus random walk with drift

$$\Delta y_t = \mu + \varepsilon_t$$

$$\Delta y_t = \alpha + \beta t + (\gamma - 1) y_{t-1} + \varepsilon_t$$

- Test whether $(\gamma - 1) = 0$
- Second equation is the same as

$$y_t = \alpha + \beta t + \gamma y_{t-1} + \varepsilon_t$$

- Table of test statistics because distribution very different than normal or other standard ones and different than with no trend

Unit root tests

Augmented Dickey-Fuller test

- Augmented Dickey-Fuller test with drift or trend
 - Trend in level versus random walk with drift

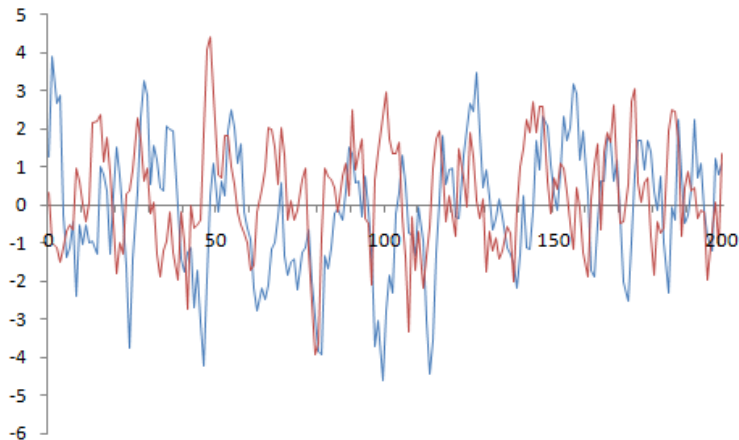
$$\Delta y_t = \mu + \sum_{i=1}^k \pi_i \Delta y_{t-i} + \varepsilon_t$$

$$\Delta y_t = \alpha + \beta t + (\gamma - 1) y_{t-1} + \sum_{i=1}^k \pi_i \Delta y_{t-i} + \varepsilon_t$$

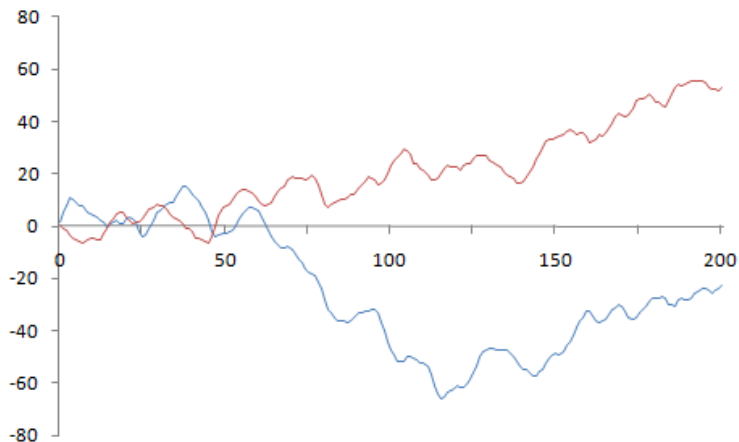
- Test whether $(\gamma - 1) = 0$
- Table of test statistics because distribution very different than normal or other standard ones
- Phillips-Perron allows for serial correlation in the error term

- Cash and futures prices for crude oil
 - They both have unit roots
 - They have to be equal at expiration
 - Is this represented in a VAR?
 - Changes in cash price and futures price in VAR
- Let's look at some simulated data

Changes in cash and futures starting off from equal levels and changes (at zero)



Levels of cash and futures prices



VARs in first differences and levels

- In a VAR with first differences of variables that have unit roots, variables can drift arbitrarily far apart
 - Nothing ties the levels of prices together

$$\begin{aligned}\Delta f_t &= \phi_{10} + \phi_{11}\Delta f_{t-1} + \phi_{12}\Delta p_{t-1} + \varepsilon_t^f \\ \Delta p_t &= \phi_{20} + \phi_{21}\Delta f_{t-1} + \phi_{22}\Delta p_{t-1} + \varepsilon_t^p\end{aligned}\quad (1)$$

- VAR in levels can tie prices together

$$\begin{aligned}f_t &= \phi_{10} + \phi_{11}f_{t-1} + \phi_{12}p_{t-1} + \varepsilon_t^f \\ p_t &= \phi_{20} + \phi_{21}f_{t-1} + \phi_{22}p_{t-1} + \varepsilon_t^p\end{aligned}\quad (2)$$

- But variables are not stationary
- Usual distributions of estimated coefficients likely to apply to equations (1)
 - But it doesn't represent what we want
- Usual distributions of estimated coefficients do not apply to equations (2)
 - But it is possible for it to represent what we want

- Want the two prices equal eventually
- Two variables are cointegrated if a linear combination of the variables has fewer unit roots than the original set of variables (e.g. $f_t - p_t$)

- Empirical strategy

- Test for unit roots in original series

- Augmented Dickey-Fuller
 - Phillips-Perron

- Test for unit roots in linear combination of variables

- Johansen test is the best generally speaking
 - If you know the value of the coefficient that should relate two variables, e.g. asset prices by coefficient of one, then an augmented Dickey-Fuller unit root test on that linear combination is better
 - Suppose f_t and p_t each have one unit root
 - Suppose $z_t = f_t - p_t$ has no unit root, but estimate $\hat{z}_t = f_t - \hat{\beta}p_t$, $\hat{\beta} \neq 1$ in general
 - Then $\hat{z}_t = f_t - \hat{\beta}p_t = z_t + (1 - \hat{\beta})p_t$, and you might infer that z_t has a unit root because \hat{z}_t does, even though z_t does not have a unit root (Why does \hat{z}_t have a unit root?)

- If variables have unit roots but are cointegrated, then there exists a Vector Error Correction Mechanism
 - Two-variable system, f_t and p_t
 - Suppose that f_t and p_t are cointegrated with $f_t - p_t$ not having a unit root
 - Ignoring cost of carry – interest on cash position
 - Cointegrating vector is $\beta' = [1 \quad -1]$
 - Therefore

$$\beta' \mathbf{y}_t = [1 \quad -1] \begin{bmatrix} f_t \\ p_t \end{bmatrix} = f_t - p_t$$

Cointegration, VARs and ECM

- It can be shown that there is a covariance stationary representation of cointegrated series
- This **cannot** be rewritten as an autoregressive representation

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

- **There does exist an Error Correction Representation**

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

- This generally is called an “Error Correction Mechanism” or “ECM” for short
- Can think of the vector autoregression in first differences as being a mis-specified equation since an error correction mechanism is the correct underlying representation of the relationship

- A two-variable, one-lag ECM is

$$\begin{aligned}f_t &= \varphi_0^1 + \alpha_1 (f_t - \beta p_t) + \phi_1^1 \Delta f_t + \phi_2^1 \Delta p_t + u_t^1 \\p_t &= \varphi_0^2 + \alpha_2 (f_t - \beta p_t) + \phi_1^2 \Delta f_t + \phi_2^2 \Delta p_t + u_t^2\end{aligned}$$

- The term $f_t - \beta p_t$ is $f_t - p_t$ if β equals one

- This Error Correction Mechanism (ECM)

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{\infty} \Phi_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

has interesting steady-state behavior

- The term $\boldsymbol{\beta}' \mathbf{y}_{t-1}$, which does not have unit root by hypothesis, forces the return of f_t and p_t to the cointegrated relationship with f_t and p_t related by $f_t - \beta p_t$ if the estimated $\boldsymbol{\alpha}$ is not equal to zero and deviations push the series back to equality

Characterization of Long-run relationship

- In the ECM

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \alpha \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{\infty} \Phi_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

- The variable $\boldsymbol{\beta}' \mathbf{y}_{t-1}$, which does not have unit root by hypothesis, makes sure that the dynamics of f_t and p_t are consistent with the long-run relationship
 - Some people would call $f - p = 0$ in this ECM an “equilibrium” relationship
 - In a statistical sense, that is reasonable
 - Without an economic model, this is not a particularly good use of the word “equilibrium”
 - A more careful use of economics would be the “stationary” or “steady state” relationship

ECM and Speed of Adjustment

- The coefficient vector α often is called the “speed of adjustment” in the ECM

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \alpha \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{\infty} \Phi_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

- The coefficient vector α is the partial derivative of $\Delta \mathbf{y}_t$ with respect to deviations from the steady state relationship $\boldsymbol{\beta}' \mathbf{y}_{t-1} = f_t - \beta p_t$
- This does not show the response of $\Delta \mathbf{y}_t$ to $\boldsymbol{\beta}' \mathbf{y}_{t-1} = (f_{t-1} - \beta p_{t-1})$ over time to a shock to \mathbf{u}_t

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ECM and Impulse Response

- The impulse response in the ECM

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

is the response to a “shock”, an “innovation” in the error term \mathbf{u}_t

- Computing the impulse response function requires iterating the equation
 - Start from steady state consistent with theory
 - Suppose that $\boldsymbol{\phi}_0 = \mathbf{0}$
 - Suppose that all variables are in the steady state, which could involve $\Delta \mathbf{y}_t$ not equal to zero but we simplify with $\Delta \mathbf{y}_t = \mathbf{0}$
 - Suppose that there is a unit shock to u_{1t} at $t = 0$, so $a_{10} = 1$
 - Then $\Delta f_0 = 1 = f_0 - f_{-1}$ and suppose $p_{-1} = 0$ for simplicity, so $f_0 = 1$
 - Then $f_0 - p_0 = 1$ and $\Delta \mathbf{y}_1$ will respond to $\boldsymbol{\beta}' \mathbf{y}_{t-1} \neq \mathbf{0}$
 - But $\Delta f_0 \neq 0$ also because $\Delta f_0 = 1$
 - Therefore the response of $\Delta \mathbf{y}_1$ to the shock to u_{10} will depend on both $\boldsymbol{\alpha}$ and $\boldsymbol{\Phi}_1$
 - The impulse response is calculated by iterating the entire equation

Computing the Impulse Response Function

- The impulse response in the ECM

$$\Delta \mathbf{y}_t = \boldsymbol{\phi}_0 + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{y}_{t-1} + \sum_{i=1}^{\infty} \boldsymbol{\Phi}_i \Delta \mathbf{y}_{t-i} + \mathbf{u}_t$$

is computed by iterating the entire equation

- The impulse response depends on all of the coefficients
- It's not hard or particularly complicated to do the underlying algebra or the computations
- It is easier to let a computer do the computations

Impulse response functions of adjusted basis for S&P 500 to a futures shock

