

# Financial Econometrics

## Market Microstructure

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February 2009

# Market microstructure

- Market microstructure is the study of the trading mechanisms used for financial securities
  - A purpose of studying microstructure is to understand and predict prices in transactions
  - People trade because they have differing values
  - Many different mechanisms are used
  - Timing
    - Continuous trading
    - Periodic auctions
  - Number of traders involved
    - Two – bargaining
    - Many – fairly competitive auctions
  - Information available to traders
    - There often is asymmetric information
    - Seller knows something buyer doesn't know
    - Buyer knows something seller doesn't know

# Roll model of transactions prices

- Dealers post bid  $b_t$  and ask  $a_t$  prices with  $b_t < a_t$ 
  - All prices in logarithms
  - Dealer buys at bid and sells at ask and pay a transaction cost of  $\tau$  for each trade
  - Customers buy at the ask price and sell at the bid price
- The behavior of the price
  - Efficient price  $p_t^* = p_{t-1}^* + a_t$  with  $E a_t = 0$  and  $\text{Var} [a_t^2] = \sigma^2$
  - Bid price  $b_t = p_t^* - \tau$
  - Ask price  $a_t = p_t^* + \tau$
  - Actual price  $p_t = p_t^* + \tau q_t$  where  $q_t = 1$  if a customer is buying and  $-1$  if a customer is selling
  - Suppose that buys and sells are equally likely and  $q_t$  is serially independent

# Bid-ask bounce in Roll model

- Not hard to show

$$\begin{aligned}\Delta p_t &= \tau q_t - \tau q_{t-1} + a_t \\ \text{Var} [\Delta p_t] &= 2\tau^2 + \sigma^2 \\ \text{Cov} [\Delta p_t, \Delta p_{t-1}] &= -\tau^2 \\ \text{Corr} [\Delta p_t, \Delta p_{t-1}] &= \frac{-\tau^2}{2\tau^2 + \sigma^2}\end{aligned}$$

- Works pretty well in practice (Hasbrouck 2004)
  - On July 2, 2003 there were 821 trades of CBL stock with an implied  $\tau$  of 1 cent per share
  - Actual spread was bigger but probably about 1.58 cents per share
- There are generalizations

# Prices recorded at irregular time intervals

- Uneven time periods raise issues different than what we have been doing
  - Trading time – time index is trades and a unit time interval is a time period
  - Calendar time
    - Information arrival process and sometimes more information, sometimes less
    - Timing of trades simply due to some stochastic process

- Prices are not a continuous variable
  - Prices on major U.S. exchanges used to be on eighths,  $1/8, 2/8, \dots$
  - Now decimals, pennies,  $.01, .02, .03$
  - Price never changes on any interval other than these units
- Sometimes this matters, sometimes not
  - Annual price changes, probably not; changes large relative to pennies and eighths
  - Transactions prices, not so obvious; sometimes yes, sometimes no
- When discreteness matters, ordered probit is a way of dealing with this

# Time pattern of trades

- Trading usually not continuous
  - Exchanges close for the night and little or no trading occurs
- Daily time-pattern of trading when open
  - Highest trading generally after open and before close
  - Less trading in the middle of trading day
- This implies a predictable change in the duration between trades during the day

# Issues with the underlying data

- Trade data are recorded for a purpose, usually for the exchange or a regulator
  - The purpose probably is not to help you with your statistical analysis
  - The data probably have issues that you have to understand to get a correct answer to your question
- Is the sequence of prices the actual sequence?
  - Depends on recording mechanism and rules
  - Usually best to go look personally at how it is done
- What do missing data mean?
  - Trades are recorded but some of the information is missing
  - Is this due to some circumstances or is it “random”?
- What if there are multiple trades within the smallest unit of time?
  - For example, multiple trades in a second when seconds is the lowest unit of time considered

- Ordered probit takes account of discreteness of prices
- Let  $\Delta p_i^* = p_{t_i}^* - p_{t_{i-1}}^*$  be the unobservable underlying price change from  $t_{i-1}$  to  $t_i$  and suppose that this price change is determined by

$$\Delta p_i^* = x_i \beta + \varepsilon_i$$

where  $x_i$  is a variable or set of variables and is conformable to  $\beta$  if they are vectors

- Let  $E \varepsilon_i = 0$ ,  $\text{Var} [\varepsilon_i | x_i] = \sigma_i^2$  and  $\text{Cov} [\varepsilon_i, \varepsilon_j | x_i] = 0$  for  $i \neq j$
- $\sigma_i^2$  is assumed to be a function of variables, including the time interval  $t_i - t_{i-1}$
- Actual price changes  $\Delta p_i$  can assume only a finite number of values, say  $s_1, s_2, \dots, s_k$

# Ordered probit estimation

- Construct the likelihood function given that  $\Delta p_i = s_j$  if  $\Delta p_i^*$  is in a range  $\alpha_{j-1} < \Delta p_i^* \leq \alpha_j$
- Estimate parameters  $\beta, \alpha_j$  and parameters in conditional likelihood function  $\text{Var}[\varepsilon_i | x_i] = \sigma_i^2$ 
  - Could set  $\alpha_j$ s to half the distance between the observable prices and not estimate them

# Decomposition approach

- Decomposition approach is another way to consider discreteness of price
- Calculate a variable based on the change in price  $\Delta p_i$

$y_i = A_i D_i S_i =$  ticks by which price changes if it changes

$A_i =$  1 if there is a change in price,  $= 0$  otherwise

$D_i | A_i =$  1 if price increases,  $= -1$  if price decreases

$S_i =$  size of price change in ticks

- A natural ordering in terms of price change and probabilities conditional on an information set  $F_{i-1}$

$$\begin{aligned}\Pr(y_i | F_{i-1}) &= \Pr(A_i D_i S_i | F_{i-1}) \\ &= \Pr(A_i | F_{i-1}) \Pr(D_i | A_i = 1, F_{i-1}) \Pr(S_i | D_i, A_i = 1, F_{i-1})\end{aligned}$$

- Set up functions for three parts with desired properties and estimate

# Autoregressive conditional duration (ACD)

- Predict intraday time between trades with models similar to ARCH models
- Let  $\Delta t_i^*$  be the time between trades adjusted for predictable daily variation
  - Typically less time between trades shortly after open and before close
  - Longer time between trades in the middle of the day
- Let  $\psi_i = E[\Delta t_i^* | F_{i-1}]$  be the conditional expectation of the time between trades conditional on information available at the time of the last trade
- Let  $\{\varepsilon_i\}$  be a sequence of iid non-negative random variables with  $E\varepsilon_i = 1$ , possibly standard exponential or standardized Weibull distribution
- Then the basic ACD( $r,s$ ) model is

$$\Delta t_i^* = \psi_i \varepsilon_i$$

$$\psi_i = \omega + \sum_{j=1}^r \gamma_j \Delta t_{i-j}^* + \sum_{j=1}^s \omega_j \psi_{i-j}$$

# Properties of the ACD model

- ACD( $r,s$ ) model

$$\Delta t_i^* = \psi_i \varepsilon_i$$

$$\psi_i = \omega + \sum_{j=1}^r \gamma_j \Delta t_{i-j}^* + \sum_{j=1}^s \omega_j \psi_{i-j}$$

- By construction

$$\begin{aligned} & \mathbb{E}[(\Delta t_i^* - \psi_i) | F_{i-1}] \\ &= \mathbb{E}[\psi_i \varepsilon_i | F_{i-1}] - \mathbb{E}[\psi_i | F_{i-1}] \\ &= \mathbb{E}[\psi_i | F_{i-1}] \mathbb{E}[\varepsilon_i] - \mathbb{E}[\psi_i | F_{i-1}] \\ &= 0 \end{aligned}$$

where I've used  $\psi_i = \mathbb{E}[\Delta t_i^* | F_{i-1}]$ ,  $\{\varepsilon_i\}$  is iid and therefore independent of  $F_{i-1}$  and  $\mathbb{E} \varepsilon_i = 1$

- Model formally can be written similar to an ARMA model

- ACD(r,s) model

$$\Delta t_i^* = \psi_i \varepsilon_i$$

$$\psi_i = \omega + \sum_{j=1}^r \gamma_j \Delta t_{i-j}^* + \sum_{j=1}^s \omega_j \psi_{i-j}$$

- Construct likelihood function and estimate by maximum likelihood
  - Similar to ARMA processes, it can be convenient to condition on the initial durations
  - Similar to ARMA processes, this estimator asymptotically has a normal distribution

# Price changes and duration

- Possibly more information when price changes
  - Tsay looks at duration between trades with changes in price in 1991
  - Now, with trading on the decimals, more likely to see price changes than when prices were on eighths or even what used to be called “teenies” (sixteenths)
  - One order may fill at several different prices

# Patterns in price changes

- I know someone who looks for patterns in nearby price changes for a successful living for a successful firm
- The firm trades on the patterns that he identifies
- What does that say about “arbitrage”?
  - Roll model implies serial correlation in prices
  - Can you profit from knowing this pattern, from knowing the model?
- What does that say about “efficiency”?

- The details of trading and trading rules are very important
  - Probably part of the reason it is called **microstructure**
  - It is easy to be lulled into complacency by the large sample sizes