

Financial Econometrics

Value at Risk

Gerald P. Dwyer, Jr.

Trinity College, Dublin

February 2009

- What do we mean by risk?
 - Dictionary: possibility of loss or injury
 - Volatility a common measure for assets
 - Two points of view on volatility measure
 - Risk is both good and bad changes
 - Volatility is useful because there is symmetry in gains and losses
- What sorts of risk?
 - Market risk
 - Credit risk
 - Liquidity risk
 - Operational risk
 - Other risks sometimes mentioned
 - Legal risk
 - Model risk

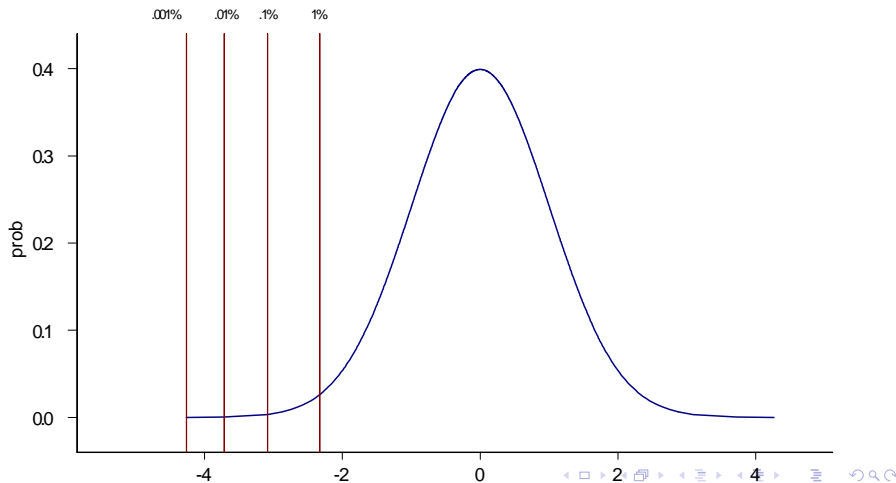
Different ways of dealing with risk

- Maximize expected utility and the preferences about risk are implicit in the utility function
 - What are problems with this?
- The worst that can happen to you
 - What are problems with this?
- Safety first
 - One definition (Roy): Investor chooses a portfolio that minimizes the probability of a loss greater in magnitude than some disaster level
 - What are problems with this?
 - Another definition (Telser): Investor specifies a maximum probability of a return less than some level and then chooses the portfolio that maximizes the expected return subject to this restriction

- Value at risk summarizes the maximum loss over some horizon with a given confidence level
 - Lower tail of distribution function for a long position
 - Upper tail of distribution function for a short position
 - Can use lower tail if symmetric
 - Suppose standard normal distribution
 - 99 percent of the time, loss is at most -2.32634
 - 1 percent of the time, loss is at least -2.32634
 - 99.99 percent of the time, loss is at most -3.71902
 - 0.01 percent of the time, loss is at least -3.71902

Illustration with a normal distribution

Lower tail of distribution



Tail of distribution

- Almost by construction, we care about unusual events, “the tail of the distribution”
- How frequently do we see these events? Suppose daily data
 - 1 percent of the time: 1 day out of every 100
 - Couple of times a year
 - 0.1 percent of the time: 1 day out of every 1,000
 - Once every four years
 - 0.01 percent of the time: 1 day out of every 10,000
 - Once every 40 years
 - 0.001 percent of the time: 1 day out of every 100,000
 - Once every 400 years
- At some point, a question arises whether the data include the risk
 - For example, 2000 to 2006 there was no financial crisis in Ireland
 - Are recent events from the same distribution?

- One interpretation of stress testing is to go far out in the tail of the distribution
 - What are some pitfalls?
- Another interpretation is to test what happens in some scenario
 - More than a little subjective

Formal definition of VaR

- Value at risk (VaR) is based on the tail of the distribution
 - Let $\Delta V(\ell)$ be the change in the value of assets over the next ℓ periods from t to $t + \ell$
 - Let $F_\ell(x)$ be the cumulative distribution function (CDF) of $\Delta V(\ell)$
 - Let p be the probability of a loss this large or larger
 - Then, for a long position with $\Delta V(\ell) < 0$ generally

$$p = \Pr(\Delta V(\ell) \leq VaR) = F_\ell(x)$$

- The loss is less than or equal to VaR with probability $1 - p$
- VaR is the p -th quintile
- Definition of quintile: For any univariate CDF $F_\ell(x)$ and probability p with $0 < p < 1$, the p -th quintile of $F_\ell(x)$ is

$$x_p = \inf \{x | F_\ell(x) \geq p\}$$

where \inf is the operator generating the smallest real number such that $F_\ell(x) \geq p$.

- VaR

$$p = \Pr(\Delta V(\ell) \leq VaR) = F_\ell(x)$$

- Suppose using a probability of 1 percent
- Suppose invest \$100 and the distribution of value changes is standard normal with zero mean
- The probability of a loss less than or equal to -2.32634 is 1 percent
- The value at risk is -2.32634 using this probability
- This is the same as the 1 percent quintile of the standard normal distribution, which is -2.32634

- Tsay presents the original RiskMetrics™ estimation strategy
 - Pretty simple IGARCH on daily data
 - One goal is to estimate relatively few parameters
 - Otherwise estimation error will overwhelm everything else
 - Another goal is to have a fairly objective strategy
 - Few or better no subjective decisions made about what parameters to include or exclude
- Technical documents available at <http://www.riskmetrics.com/publications/techdoc.html>
- Problems with IGARCH
 - There is “long memory” in volatility
 - Autocorrelations of absolute value of returns do not decrease exponentially as indicated for linear and ARCH systems

- LM-ARCH – long memory ARCH
- Variances at different time scales used and weighted with exponential decay
 - Mean return not zero, especially for stocks and bonds
 - Quantitatively small effects but introduce clear deviations from forecasted volatilities
 - Use autoregressive components from the last two years and estimate mean return over last two years
 - I will suppress this
 - IGARCH(1,1) is

$$\begin{aligned}r_t &= \sigma_t \varepsilon_t, & \mathbb{E} \varepsilon_t &= 0, & \mathbb{E} \varepsilon_t^2 &= 1 \\ \sigma_t^2 &= \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \\ 0 &< \beta_1 < 1\end{aligned}$$

- New RiskMetrics setup is rather more complicated

$$\begin{aligned}r_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \sum_{k=1}^{k_{\max}} w_k \sigma_{k,t}^2 \\ w_k &= \frac{1}{C} \left(1 - \frac{\ln \tau_k}{\ln(\tau_0)} \right) \\ \tau_k &= \tau_1 \rho^{k-1} \quad k = 1, \dots, k_{\max} \\ \sigma_{k,t}^2 &= \mu_k \sigma_{k,t-1}^2 + (1 - \mu_k) r_t^2 \\ \mu_k &= \exp(-1/\tau_k)\end{aligned}$$

- Pages 8 and 9 of long document
 - $\tau_k = \tau_1 \rho^{k-1}$ determines weights
 - There is an underlying IGARCH(1,1) process with the weight determined by $\mu_k = \exp(-1/\tau_k)$

- Tsay spends a fair amount of time on an approach he suggests, extreme value theory
 - Behavior of tail of distribution determines limiting behavior of minimum
 - Interesting